

A Microscopic Transport Framework for Heavy-Ion Collisions from High to Low Energies — PHSD/PHQMD

Jiaxing Zhao (赵佳星)

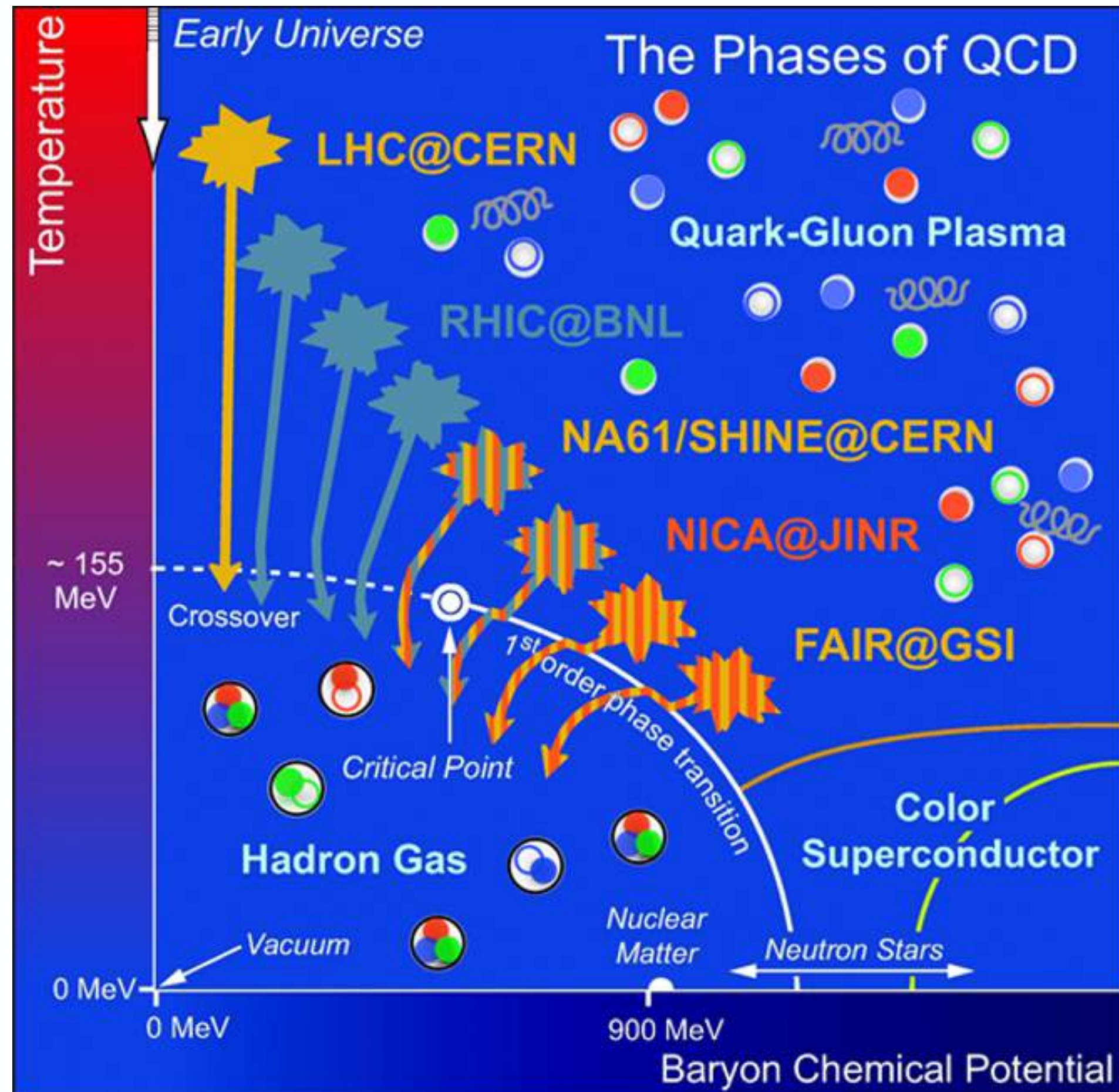
**Helmholtz Research Academy Hessen for FAIR (HFHF)
& Goethe University**

Thanks to: Elena Bratkovskaya

2025-12-11

Heavy ion collisions

QCD phase diagram in the (T, μ_b) plane



- **Equation-of-State** of hot and dense matter;
- Study of the **phase transition** from hadronic to Quark-Gluon Plasma (QGP);
- Search for a critical end point (**CEP**);
- Search for signatures of **chiral symmetry restoration**;
- Study of the **in-medium properties** of hadrons at high baryon density and temperature;
- ...

Dynamical description of Heavy ion collisions



The goal:

to study the properties of **strongly interacting matter** under extreme conditions from **a microscopic point of view**

to develop a dynamical microscopic transport approach for strongly interacting systems



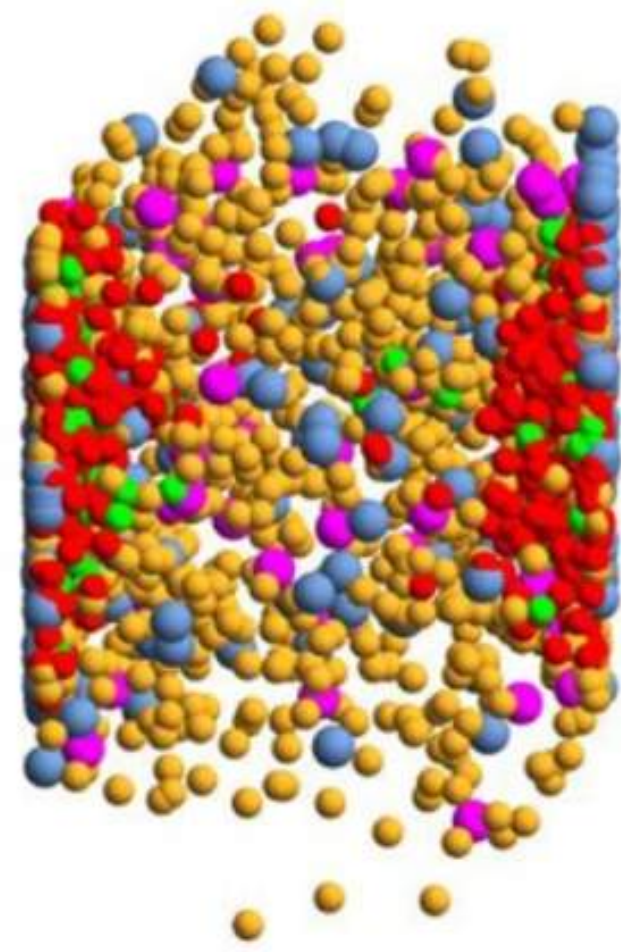
➔ PHSD (**P**arton-**H**adron-**S**tring-**D**ynamics)

➔ PHQMD(**P**arton-**H**adron-**Q**uantum-**M**olecular-**D**ynamics)

PHSD:



I. Development fo the microscopic transport theory: from BUU to Kadanoff-Baym dynamics



History: semi-classical BUU equation

Boltzmann-Uehling-Uhlenbeck equation (non-relativistic formulation) — propagation of particles in the self-generated Hartree-Fock mean-field potential $U(r,t)$ with an **on-shell collision term**:

$$\frac{\partial}{\partial t} f(\vec{r}, \vec{p}, t) + \frac{\vec{p}}{m} \vec{\nabla}_{\vec{r}} f(\vec{r}, \vec{p}, t) - \vec{\nabla}_{\vec{r}} U(\vec{r}, t) \vec{\nabla}_{\vec{p}} f(\vec{r}, \vec{p}, t) = \left(\frac{\partial f}{\partial t} \right)_{coll}$$

collision term:
elastic and inelastic reactions



$f(\vec{r}, \vec{p}, t)$ is the **single particle phase-space distribution function** — probability to find the particle at position r with momentum p at time t

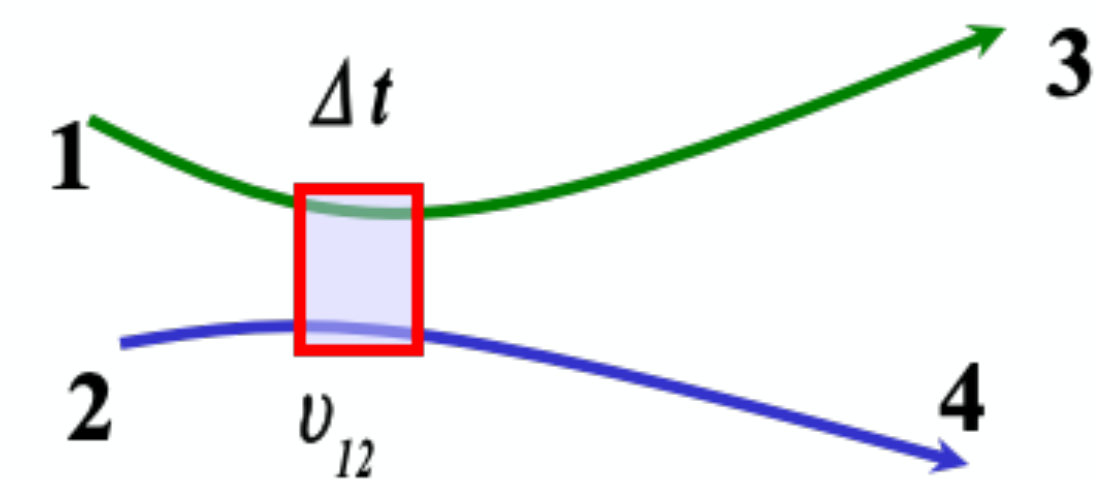
□ self-generated Hartree-Fock mean-field potential:

$$U(\vec{r}, t) = \frac{1}{(2\pi\hbar)^3} \sum_{\beta_{occ}} \int d^3 r' d^3 p V(\vec{r} - \vec{r}', t) f(\vec{r}', \vec{p}, t) + (\text{Fock term})$$

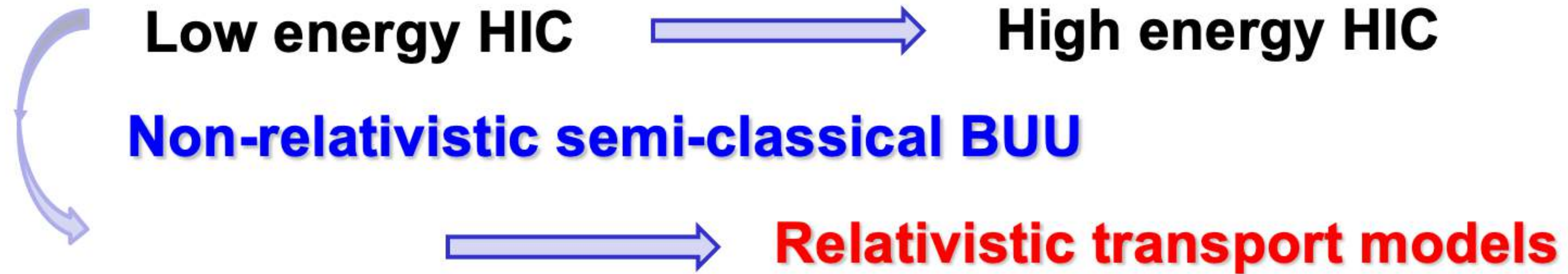
□ **Collision term** for $1+2 \rightarrow 3+4$ (let's consider fermions) :

$$I_{coll} = \frac{4}{(2\pi)^3} \int d^3 p_2 d^3 p_3 \int d\Omega |v_{12}| \delta^3(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4) \cdot \frac{d\sigma}{d\Omega}(1+2 \rightarrow 3+4) \cdot P$$

Including Pauli blocking: $P = \underbrace{f_3 f_4 (1 - f_1)(1 - f_2)}_{\text{Gain term: } 3+4 \rightarrow 1+2} - \underbrace{f_1 f_2 (1 - f_3)(1 - f_4)}_{\text{Loss term: } 1+2 \rightarrow 3+4}$



History: developments of relativistic transport models



‘Relativistic Vlasov-Uehling-Uhlenbeck model for heavy-ion collisions’
Che-Ming Ko, Qi Li, Phys.Rev. C37 (1988) 2270

‘Covariant Boltzmann-Uehling-Uhlenbeck approach for heavy-ion collisions’
Bernhard Blaettel, Volker Koch, Wolfgang Cassing, Ulrich Mosel, Phys.Rev. C38 (1988) 1767;

‘Relativistic BUU approach with momentum dependent mean fields’
T. Maruyama, B. Blaettel, W. Cassing, A. Lang, U. Mosel, K. Weber, Phys.Lett. B297 (1992) 228

‘The Relativistic Landau-Vlasov method in heavy ion collisions’
C. Fuchs, H.H. Wolter, Nucl.Phys. A589 (1995) 732

....

DJBUU, GiBUU, IBL, IBUU, LBUU, pBUU, RBUU, RVUU, SMASH, BAMPS,...

Covariant transport equation

□ Covariant relativistic on-shell BUU equation :

from many-body theory by connected Green functions in phase-space + mean-field limit for the propagation part (VUU)

$$\left\{ \left(\Pi_\mu - \Pi_\nu (\partial_\mu^p U_\nu^v) - m^* (\partial_\mu^p U_S^v) \right) \partial_x^\mu + \left(\Pi_\nu (\partial_\mu^x U_\nu^v) + m^* (\partial_\mu^x U_S^v) \right) \partial_p^\mu \right\} f(\mathbf{x}, \mathbf{p}) = I_{coll}$$

$$I_{coll} \equiv \sum_{2,3,4} \int d2 d3 d4 [G^+ G]_{1+2 \rightarrow 3+4} \delta^4(\Pi + \Pi_2 - \Pi_3 - \Pi_4)$$

$$d2 \equiv \frac{d^3 p_2}{E_2}$$

$$\times \{ f(\mathbf{x}, \mathbf{p}_3) f(\mathbf{x}, \mathbf{p}_4) (1 - f(\mathbf{x}, \mathbf{p})) (1 - f(\mathbf{x}, \mathbf{p}_2))$$

Gain term
3+4 → 1+2

$$- f(\mathbf{x}, \mathbf{p}) f(\mathbf{x}, \mathbf{p}_2) (1 - f(\mathbf{x}, \mathbf{p}_3)) (1 - f(\mathbf{x}, \mathbf{p}_4)) \}$$

Loss term
1+2 → 3+4

where $\partial_\mu^x \equiv (\partial_t, \vec{\nabla}_r)$

$$m^*(\mathbf{x}, \mathbf{p}) = m + U_S(\mathbf{x}, \mathbf{p}) \quad - \text{effective mass}$$

$$\Pi_\mu(\mathbf{x}, \mathbf{p}) = p_\mu - U_\mu(\mathbf{x}, \mathbf{p}) \quad - \text{effective momentum}$$

W. Ehehalt, W. Cassing, Nucl. Phys. A 602 (1996) 449

$U_S(\mathbf{x}, \mathbf{p}), U_\mu(\mathbf{x}, \mathbf{p})$ are scalar and vector part of particle **self-energies**

$\delta(\Pi_\mu \Pi^\mu - m^{*2})$ – mass-shell constraint

Dynamical transport model: collision terms

□ BUU eq. for different particles of type $i=1,\dots,n$

Hadronic transport models: BUU, IQMD, UrQMD, GiBUU, HSD, JAM, SMASH, ...

$$Df_i \equiv \frac{d}{dt} f_i = I_{coll} [f_1, f_2, \dots, f_n]$$

Drift term=Vlasov eq.

collision term

i : *Baryons* : $p, n, \Delta(1232), N(1440), N(1535), \dots, \Lambda, \Sigma, \Sigma^*, \Xi, \Omega; \Lambda_c$

Mesons : $\pi, \eta, K, \bar{K}, \rho, \omega, K^*, \eta', \phi, a_1, \dots, D, \bar{D}, J/\Psi, \Psi', \dots$

→ **coupled set of BUU equations** for different particles of type $i=1,\dots,n$

$$\left\{ \begin{array}{l} Df_N = I_{coll} [f_N, f_\Delta, f_{N(1440)}, \dots, f_\pi, f_\rho, \dots] \\ Df_\Delta = I_{coll} [f_N, f_\Delta, f_{N(1440)}, \dots, f_\pi, f_\rho, \dots] \\ \dots \\ Df_\pi = I_{coll} [f_N, f_\Delta, f_{N(1440)}, \dots, f_\pi, f_\rho, \dots] \\ \dots \end{array} \right.$$

From weakly to strongly interacting systems

Properties of matter (on hadronic and partonic levels) in heavy-ion collisions: **QGP** – strongly interacting system! Degrees of freedom – dressed partons **Hadronic matter** – in-medium effects – modification of hadron properties at finite T, μ_B (vector mesons, strange mesons)

Many-body theory:

Strong interaction → large width = short life-time
→ broad spectral function → **quantum object**

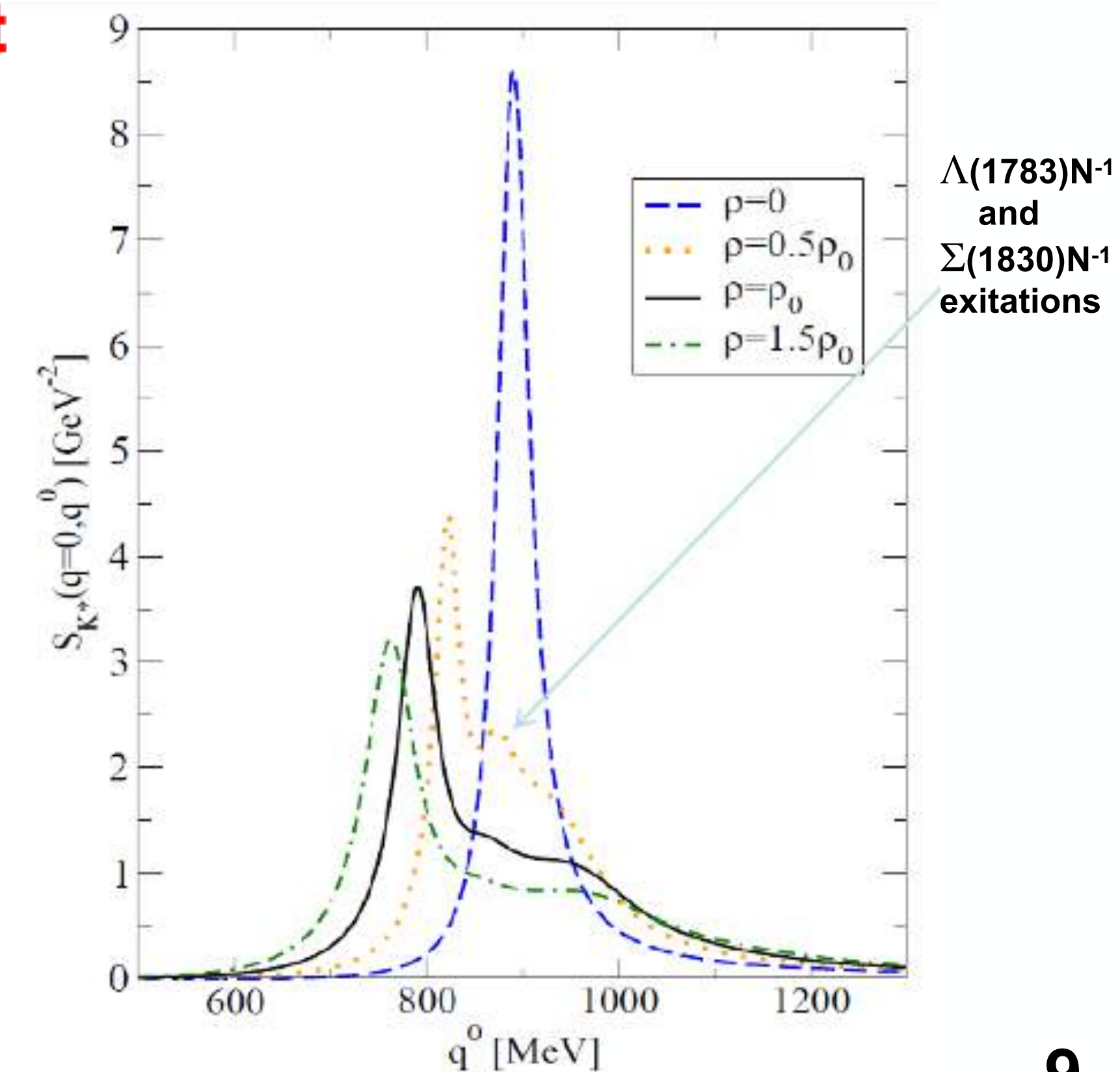
■ How to describe the dynamics of broad **strongly interacting quantum states** in transport theory?

□ semi-classical BUU

first order gradient expansion of quantum **Kadanoff-Baym equations**

□ **generalized transport equations based on Kadanoff-Baym dynamics**

Kbar* spectral function



Dynamical description of strongly interacting systems

Quantum field theory →

Kadanoff-Baym dynamics for resummed single-particle Green functions $S^<$

$$\hat{S}_{0x}^{-1} S_{xy}^< = \sum_{xz}^{ret} \odot S_{zy}^< + \sum_{xz}^< \odot S_{zy}^{adv}$$

Integration over the intermediate spacetime

Green functions $S^<$ / self-energies Σ :

$$iS_{xy}^< = \eta \langle \{ \Phi^+(y) \Phi(x) \} \rangle$$

$$iS_{xy}^> = \langle \{ \Phi(y) \Phi^+(x) \} \rangle$$

$$iS_{xy}^c = \langle T^c \{ \Phi(x) \Phi^+(y) \} \rangle \text{ - causal}$$

$$iS_{xy}^a = \langle T^a \{ \Phi(x) \Phi^+(y) \} \rangle \text{ - anticausal}$$

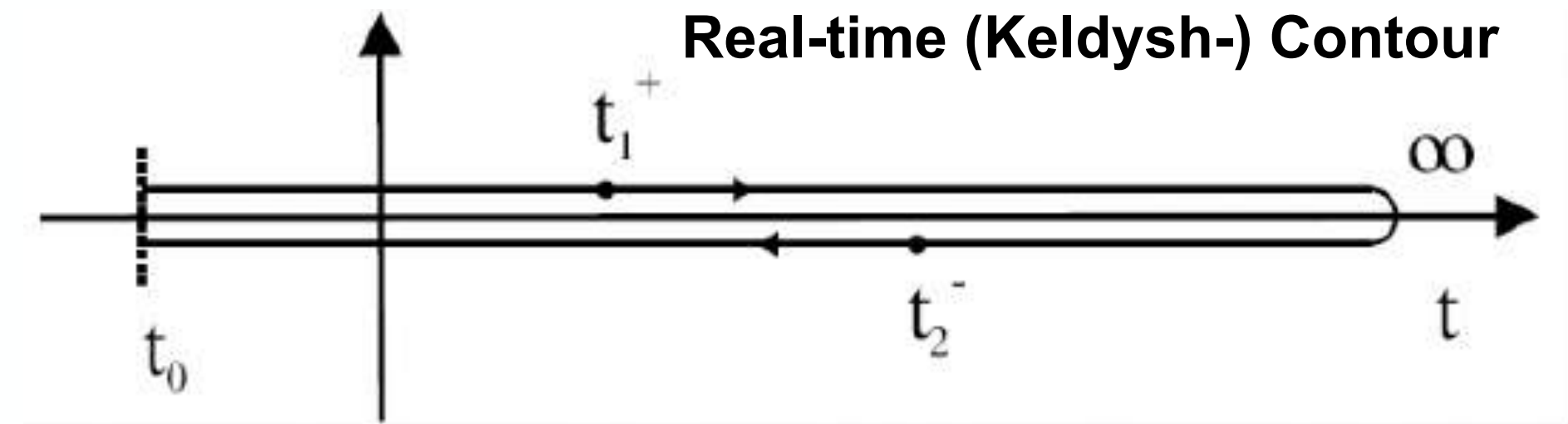
$$S_{xy}^{ret} = S_{xy}^c - S_{xy}^< = S_{xy}^> - S_{xy}^a \text{ - retarded}$$

$$S_{xy}^{adv} = S_{xy}^c - S_{xy}^> = S_{xy}^< - S_{xy}^a \text{ - advanced}$$

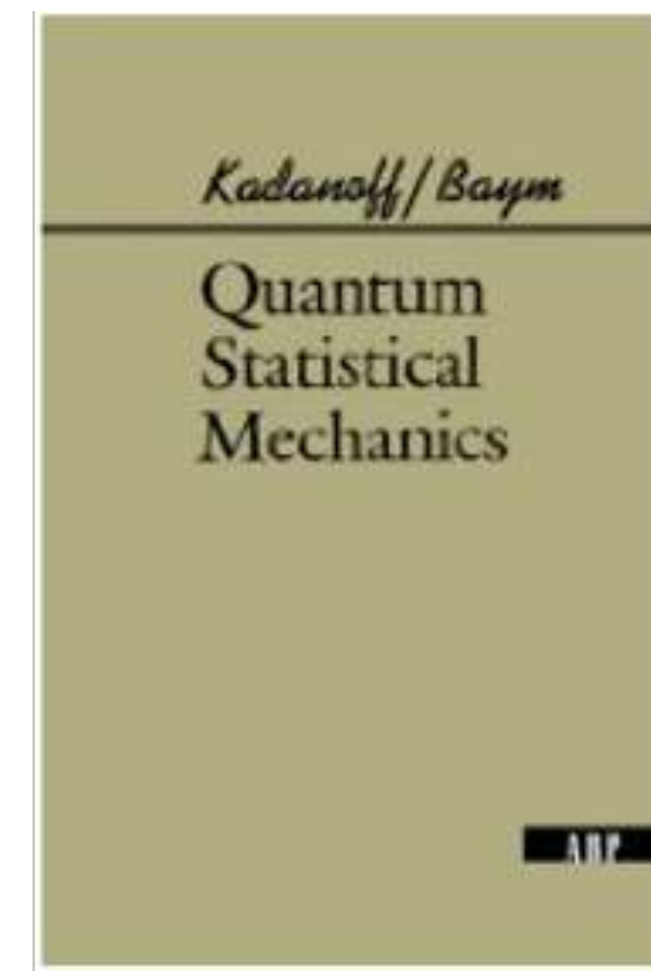
$$\hat{S}_{0x}^{-1} \equiv -(\partial_x^\mu \partial_\mu^x + M_0^2)$$

$$\eta = \pm 1 (\text{bosons / fermions})$$

$T^a (T^c)$ - (anti-)time - ordering operator



Leo Kadanoff



Gordon Baym

From Kadanoff-Baym Eq. to generalized transport Eq.

After the first-order gradient expansion of the Wigner transformed Kadanoff-Baym equations and separation into the real and imaginary parts one gets:

$$\diamond \{F_1\} \{F_2\} := \frac{1}{2} \left(\frac{\partial F_1}{\partial X_\mu} \frac{\partial F_2}{\partial P^\mu} - \frac{\partial F_1}{\partial P_\mu} \frac{\partial F_2}{\partial X^\mu} \right)$$

Generalized transport equations (GTE):

$$\begin{array}{cccc} \text{drift term} & \text{Vlasov term} & \text{backflow term} & \text{collision term = ,gain' - ,loss' term} \\ \diamond \{ P^2 - M_0^2 - \text{Re} \Sigma_{XP}^{\text{ret}} \} \{ S_{XP}^< \} - & \diamond \{ \Sigma_{XP}^< \} \{ \text{Re} S_{XP}^{\text{ret}} \} & = & \frac{i}{2} [\Sigma_{XP}^> S_{XP}^< - \Sigma_{XP}^< S_{XP}^>] \end{array}$$

Backflow term incorporates the **off-shell** behavior in the particle propagation
! vanishes in the quasiparticle limit $A_{XP} \rightarrow \delta(p^2 - M^2)$

□ Propagation of the Green function $iS_{XP}^< = A_{XP} N_{XP}$, which carries information not only on the **number of particles** (N_{XP}), but also on their **properties**, interactions and correlations (via A_{XP})

□ **Spectral function:** $A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - \text{Re} \Sigma_{XP}^{\text{ret}})^2 + \Gamma_{XP}^2/4}$

W. Botermans, R. Malfliet, Phys. Rep. 198 (1990) 115

Reaction rate of particle (at space-time position **X**): $\Gamma_{XP} = -\text{Im} \Sigma_{XP}^{\text{ret}} = 2p_0 \Gamma$
 where Γ is the **width of spectral function**

Generalized testparticle off-shell equations of motion

- Employ **testparticle Ansatz** for the real valued quantity $iS_{XP}^<$

$$F_{XP} = A_{XP}N_{XP} = iS_{XP}^< \sim \sum_{i=1}^N \delta^{(3)}(\vec{X} - \vec{X}_i(t)) \delta^{(3)}(\vec{P} - \vec{P}_i(t)) \delta(P_0 - \epsilon_i(t))$$

insert in generalized transport equations and determine **equations of motion** !

→ Generalized testparticle Cassing-Juchem off-shell equations of motion for the time-like particles:

$$\frac{d\vec{X}_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[2\vec{P}_i + \vec{\nabla}_{P_i} \text{Re}\Sigma_{(i)}^{\text{ret}} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\Gamma_{(i)}} \vec{\nabla}_{P_i} \Gamma_{(i)} \right],$$

$$\frac{d\vec{P}_i}{dt} = -\frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\vec{\nabla}_{X_i} \text{Re}\Sigma_{(i)}^{\text{ret}} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\Gamma_{(i)}} \vec{\nabla}_{X_i} \Gamma_{(i)} \right],$$

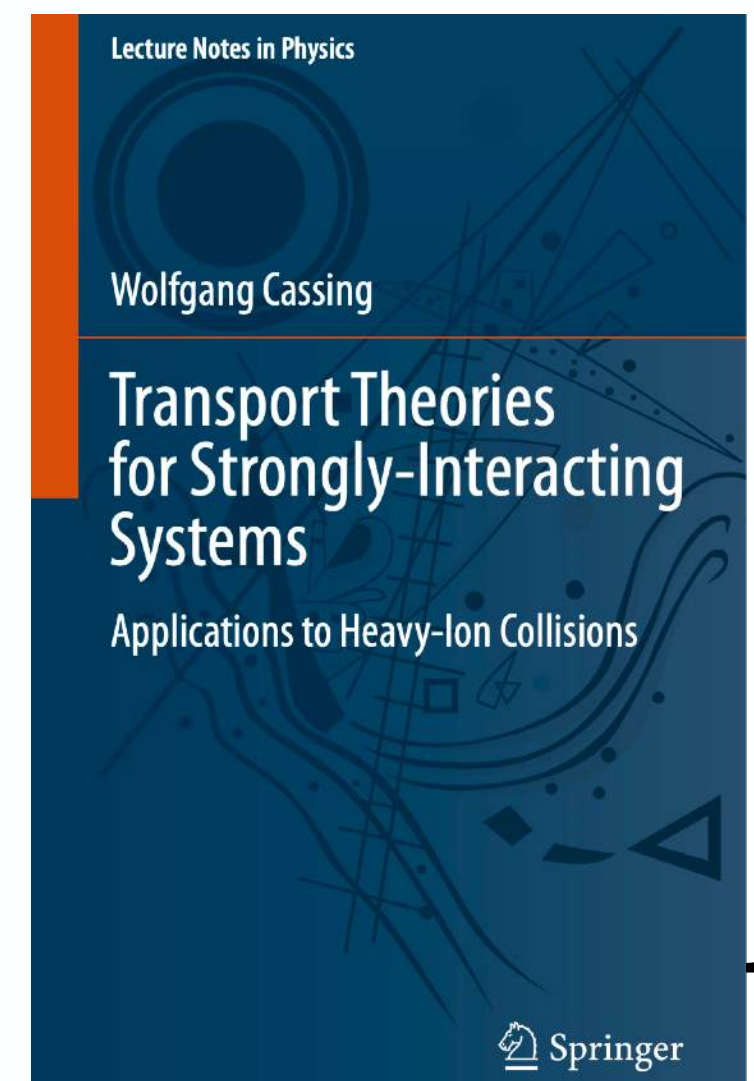
$$\frac{d\epsilon_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\frac{\partial \text{Re}\Sigma_{(i)}^{\text{ret}}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\Gamma_{(i)}} \frac{\partial \Gamma_{(i)}}{\partial t} \right],$$

with $F_{(i)} \equiv F(t, \vec{X}_i(t), \vec{P}_i(t), \epsilon_i(t))$

$$C_{(i)} = \frac{1}{2\epsilon_i} \left[\frac{\partial}{\partial \epsilon_i} \text{Re}\Sigma_{(i)}^{\text{ret}} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\Gamma_{(i)}} \frac{\partial}{\partial \epsilon_i} \Gamma_{(i)} \right]$$

W. Cassing , S. Juchem, NPA 665 (2000) 377; 672 (2000) 417; 677 (2000) 445

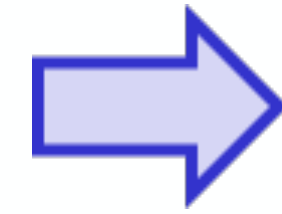
Realized in PHSD



On-shell limits: from KB to BUU

□ $\Gamma(X,P) \rightarrow 0$

$$A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - \text{Re}\Sigma_{XP}^{\text{ret}})^2 + \Gamma_{XP}^2/4}$$



quasiparticle approximation:

$$A_{XP} = 2p\delta(P^2 - M_0^2)$$

□ $\Gamma(X,P)$ such that

E.g.: $\Gamma = \text{const}$ or $\Gamma = \Gamma_{\text{vacuum}}(M)$

Vacuum spectral function with constant or mass dependent width Γ :

i.e. spectral function A_{XP} does **NOT change the shape** (and pole position) during propagation through the medium (no density-, T-dependence)

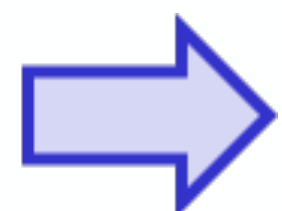
In on-shell limits the “**backflow term**” - which incorporates the off-shell behavior in the particle propagation - **vanishes**:

$$\frac{d\vec{X}_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[2\vec{P}_i + \vec{\nabla}_{P_i} \text{Re}\Sigma_{(i)}^{\text{ret}} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\Gamma_{(i)}} \vec{\nabla}_{P_i} \Gamma_{(i)} \right],$$

$$\frac{d\vec{P}_i}{dt} = -\frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\vec{\nabla}_{X_i} \text{Re}\Sigma_{(i)}^{\text{ret}} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\Gamma_{(i)}} \vec{\nabla}_{X_i} \Gamma_{(i)} \right],$$

$$\frac{d\epsilon_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\frac{\partial \text{Re}\Sigma_{(i)}^{\text{ret}}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\Gamma_{(i)}} \frac{\partial \Gamma_{(i)}}{\partial t} \right],$$

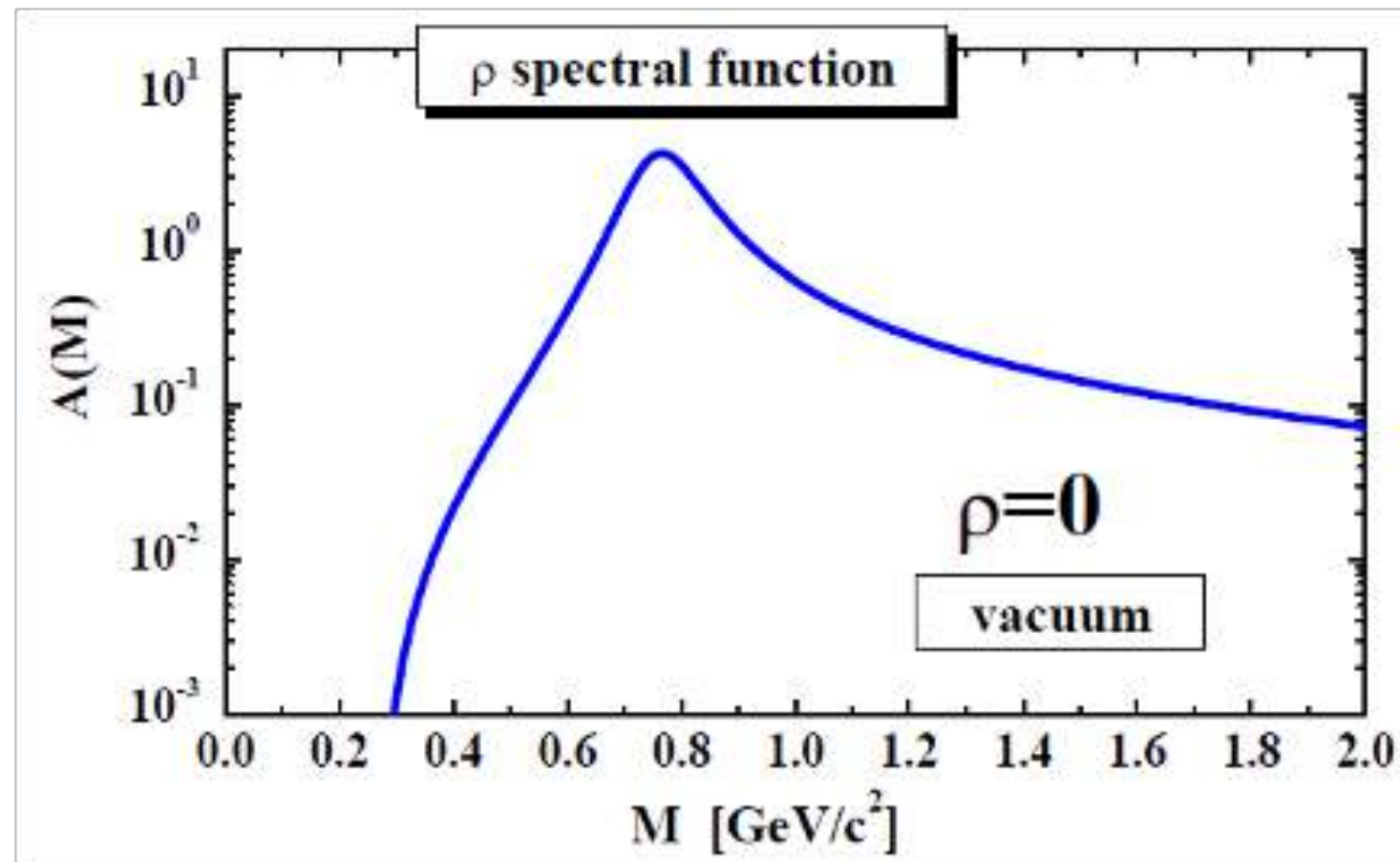
W. Cassing, Eur. Phys. J. ST 168 (2009) 3



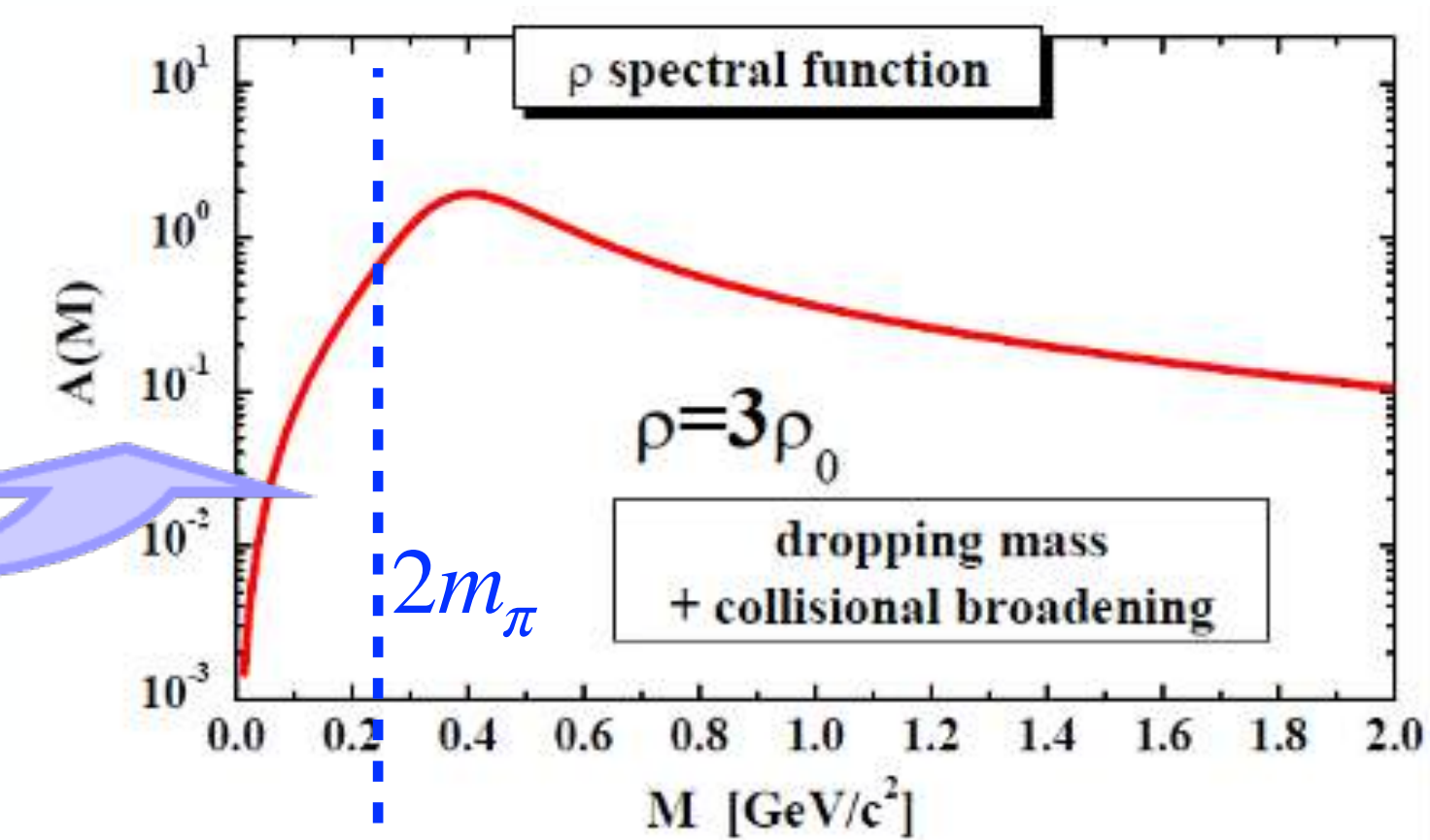
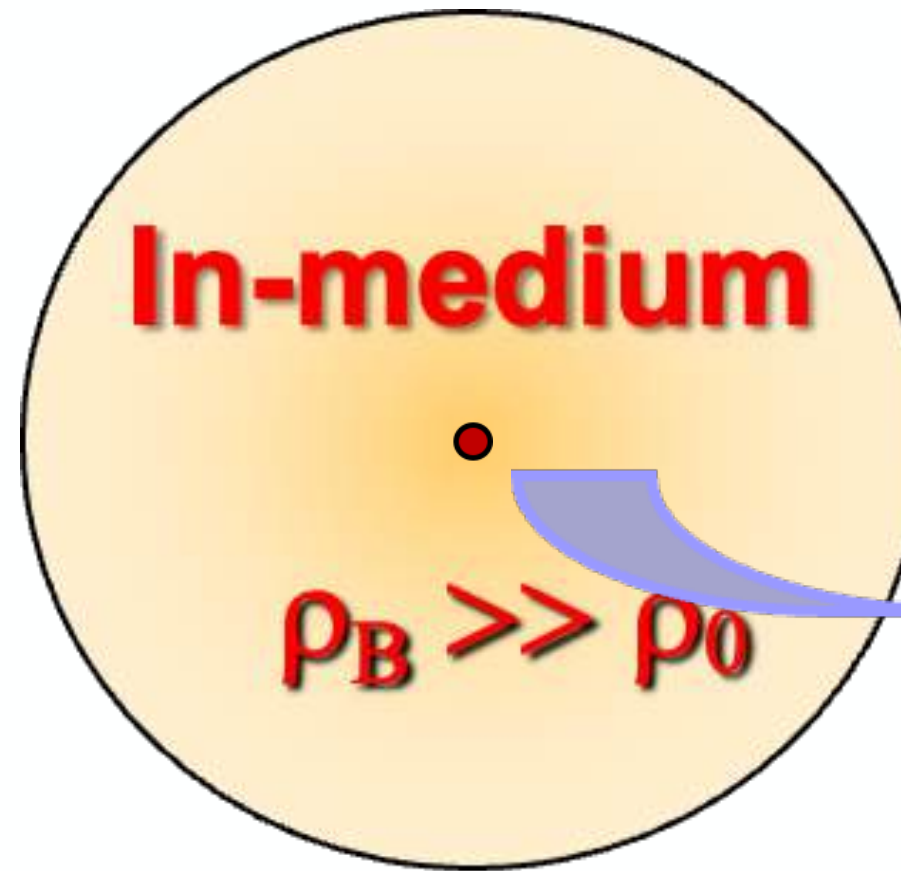
Hamilton equations of motion (independent on Γ) \rightarrow **BUU limit**

Off-shell vs. on-shell transport dynamics

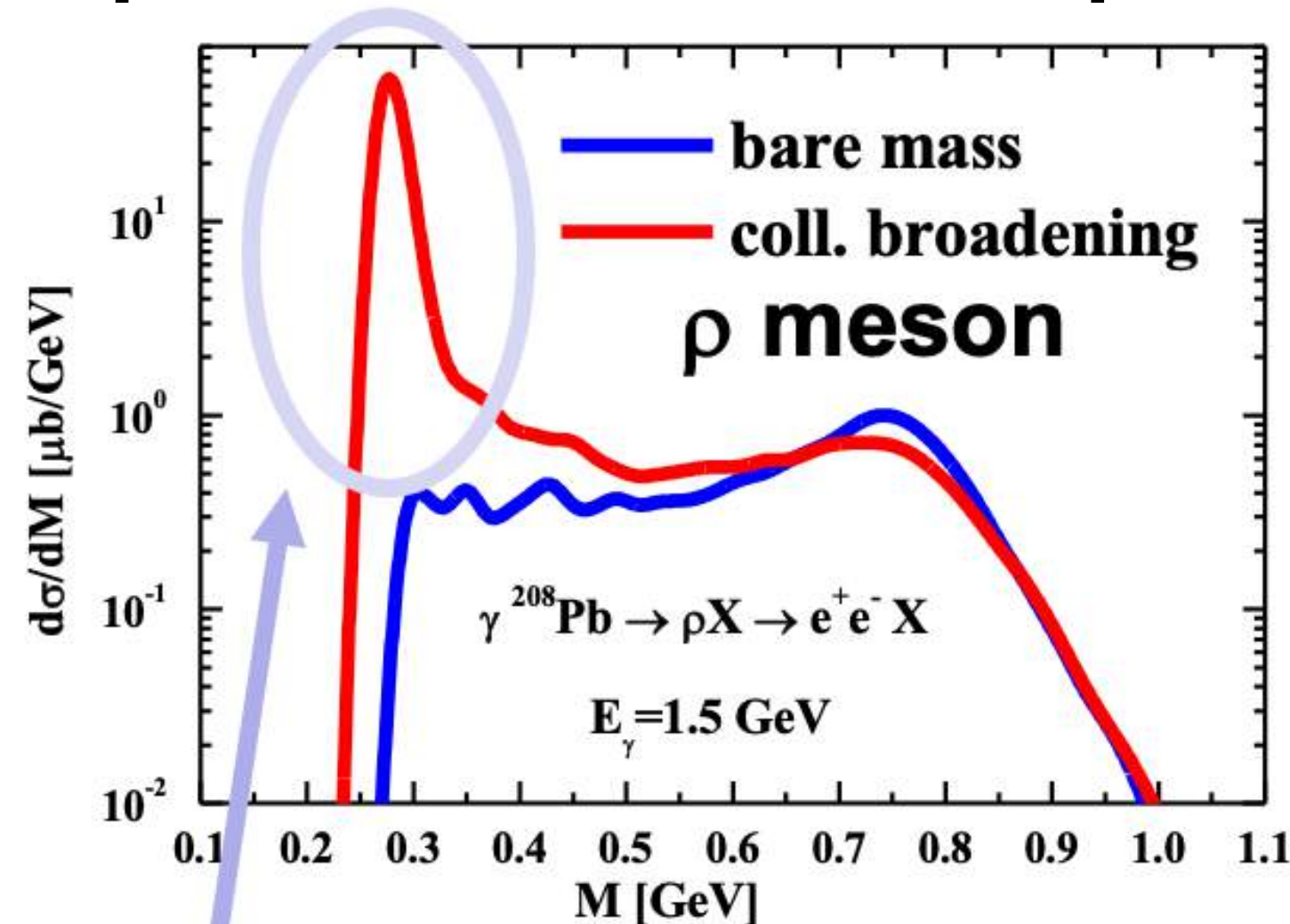
Vacuum state ($\rho = 0$)



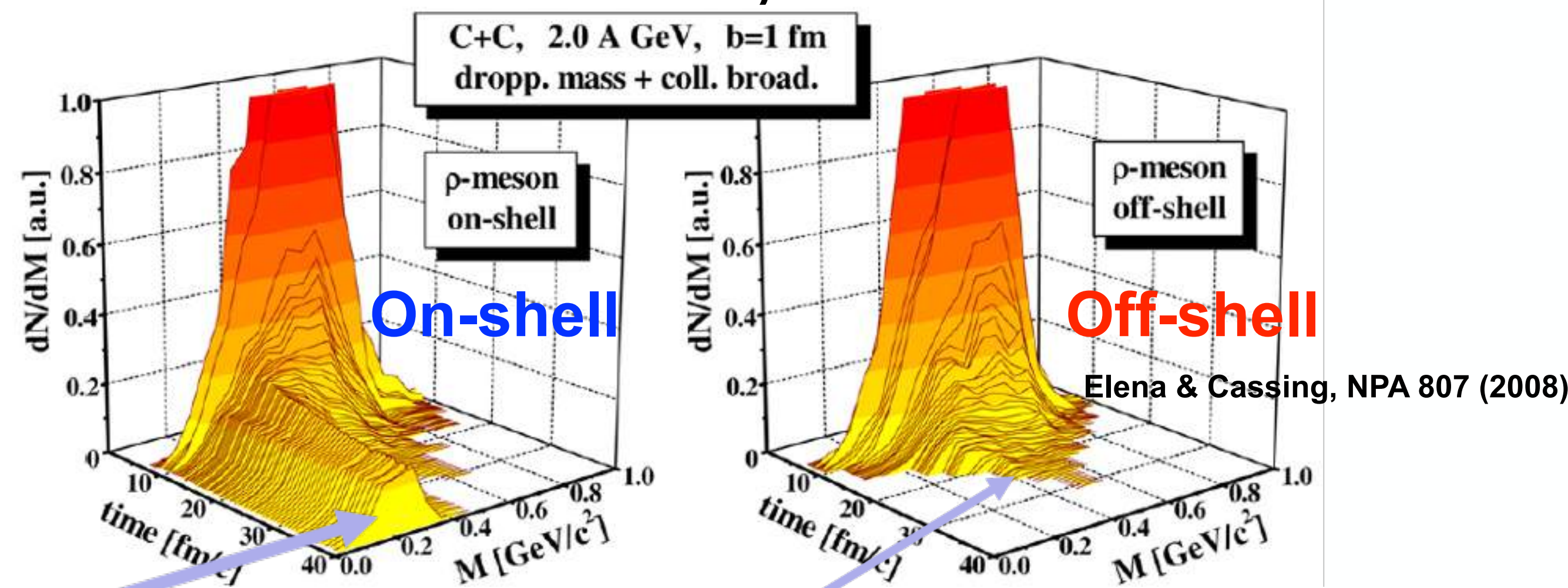
In-medium: production of broad state



Dilepton invariant mass spectra



Mass distribution of ρ mesons vs time



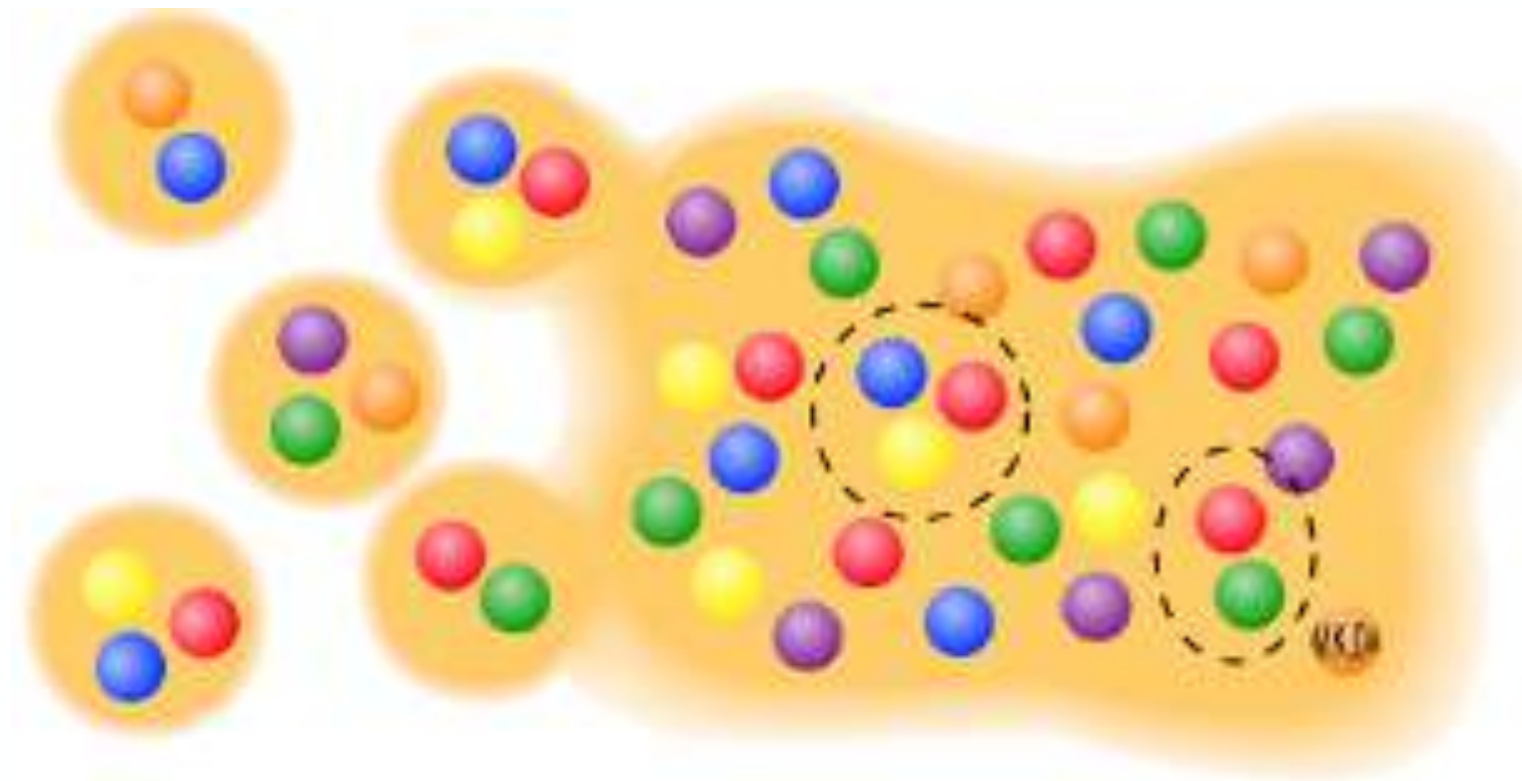
low mass ρ and ω mesons live forever (and shine “fake” dileptons)

The off-shell spectral function becomes on-shell in the vacuum **dynamically** by propagation!14

PHSD:



II. Modeling of sQGP for high energy nuclear collisions in microscopic transport theory



Description of the partonic and hadronic phase

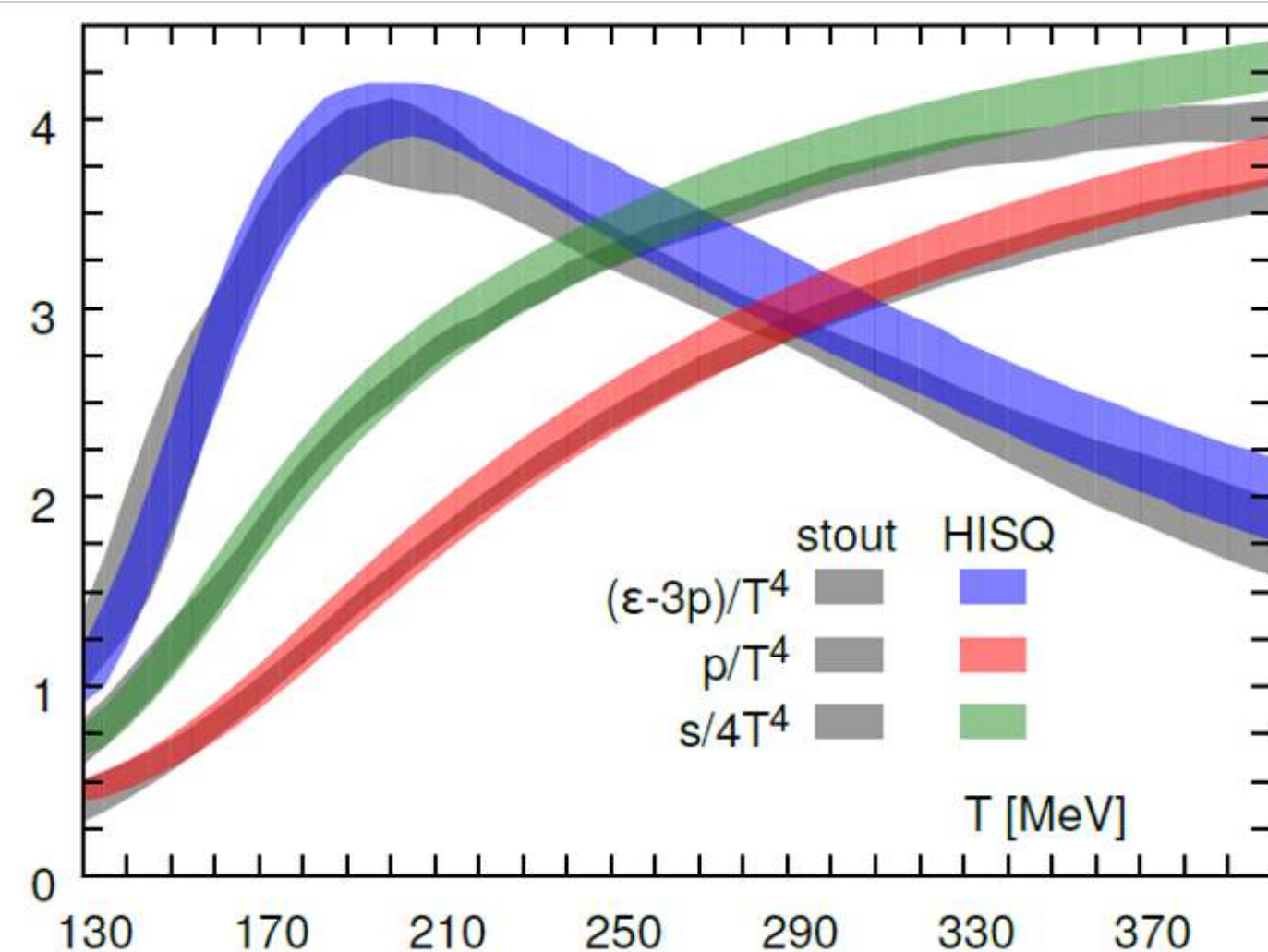
- How to model a **QGP phase** in line with IQCD data?
- How to solve the hadronization problem?

Hydro based models: MUSIC, VISHNU, CLVisc, vHLLE, ...

pQCD based models: BAMPS, AMPT, ...

Thermal QCD based model: PHSD

Macroscopic (EoS) \rightarrow microscopic parton properties (mass, interactions, ...)



Strongly coupling QGP

\rightarrow a strongly interacting quasiparticle system

Dynamical QuasiParticle Model (DQPM)

DQPM – effective model for the description of **non-perturbative** (strongly interacting) QCD based on **IQCD EoS**

Theoretical basis: resummed single-particle Green's functions → quark (gluon) propagator (2PI)

$$\begin{aligned} \text{gluon propagator: } \Delta^{-1} &= P^2 - \Pi & \& \quad \text{quark propagator } S_q^{-1} &= P^2 - \Sigma_q \\ \text{gluon self-energy: } \Pi &= M_g^2 - i2\gamma_g\omega & \& \quad \text{quark self-energy: } \Sigma_q &= M_q^2 - i2\gamma_q\omega \end{aligned}$$

Modeling of the quark/gluon **masses** and **widths** (ansatz inspired by HTL calculations)

Masses:

$$M_{q(\bar{q})}^2(T, \mu_B) = \frac{N_c^2 - 1}{8N_c} g^2(T, \mu_B) \left(T^2 + \frac{\mu_q^2}{\pi^2} \right)$$

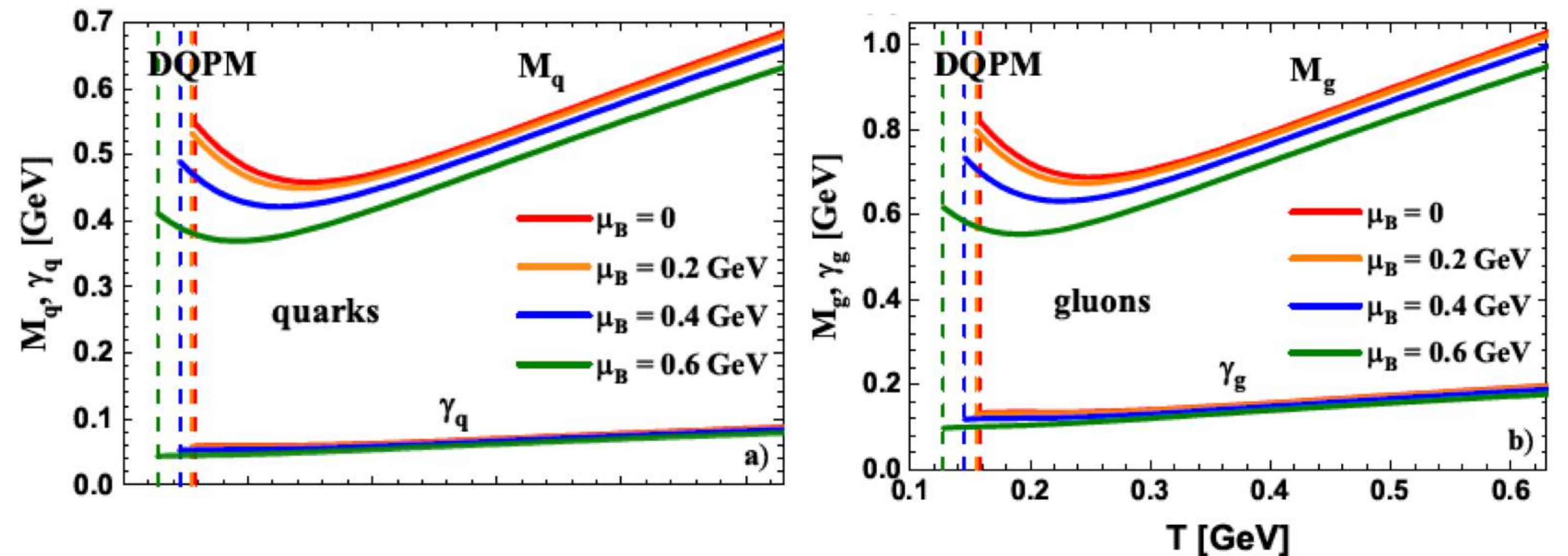
$$M_g^2(T, \mu_B) = \frac{g^2(T, \mu_B)}{6} \left(\left(N_c + \frac{1}{2} N_f \right) T^2 + \frac{N_c}{2} \sum_q \frac{\mu_q^2}{\pi^2} \right)$$

Widths:

$$\gamma_{q(\bar{q})}(T, \mu_B) = \frac{1}{3} \frac{N_c^2 - 1}{2N_c} \frac{g^2(T, \mu_B) T}{8\pi} \ln \left(\frac{2c}{g^2(T, \mu_B)} + 1 \right)$$

$$\gamma_g(T, \mu_B) = \frac{1}{3} N_c \frac{g^2(T, \mu_B) T}{8\pi} \ln \left(\frac{2c}{g^2(T, \mu_B)} + 1 \right)$$

→ (T, μ_B) dependent **coupling constant** can be determined from lattice results



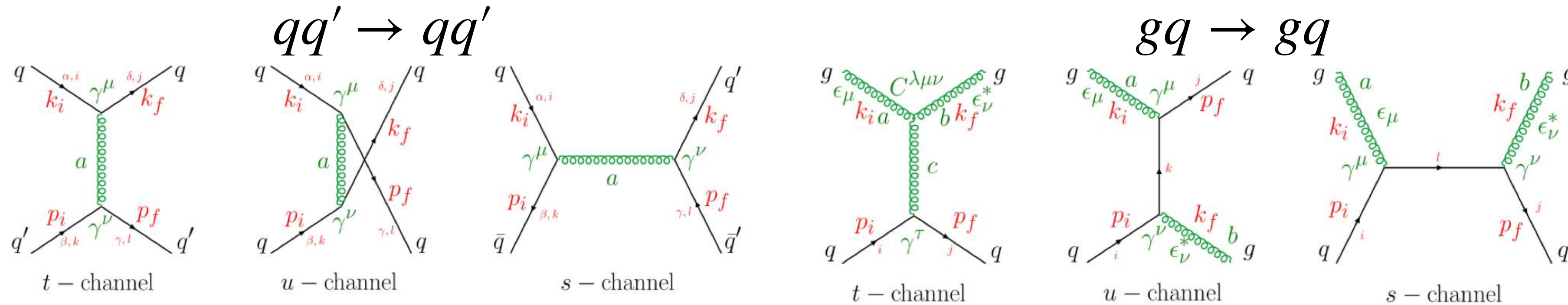
ML method (Olga; LongGang): without any ansatz ! 17

Partonic interactions: matrix elements

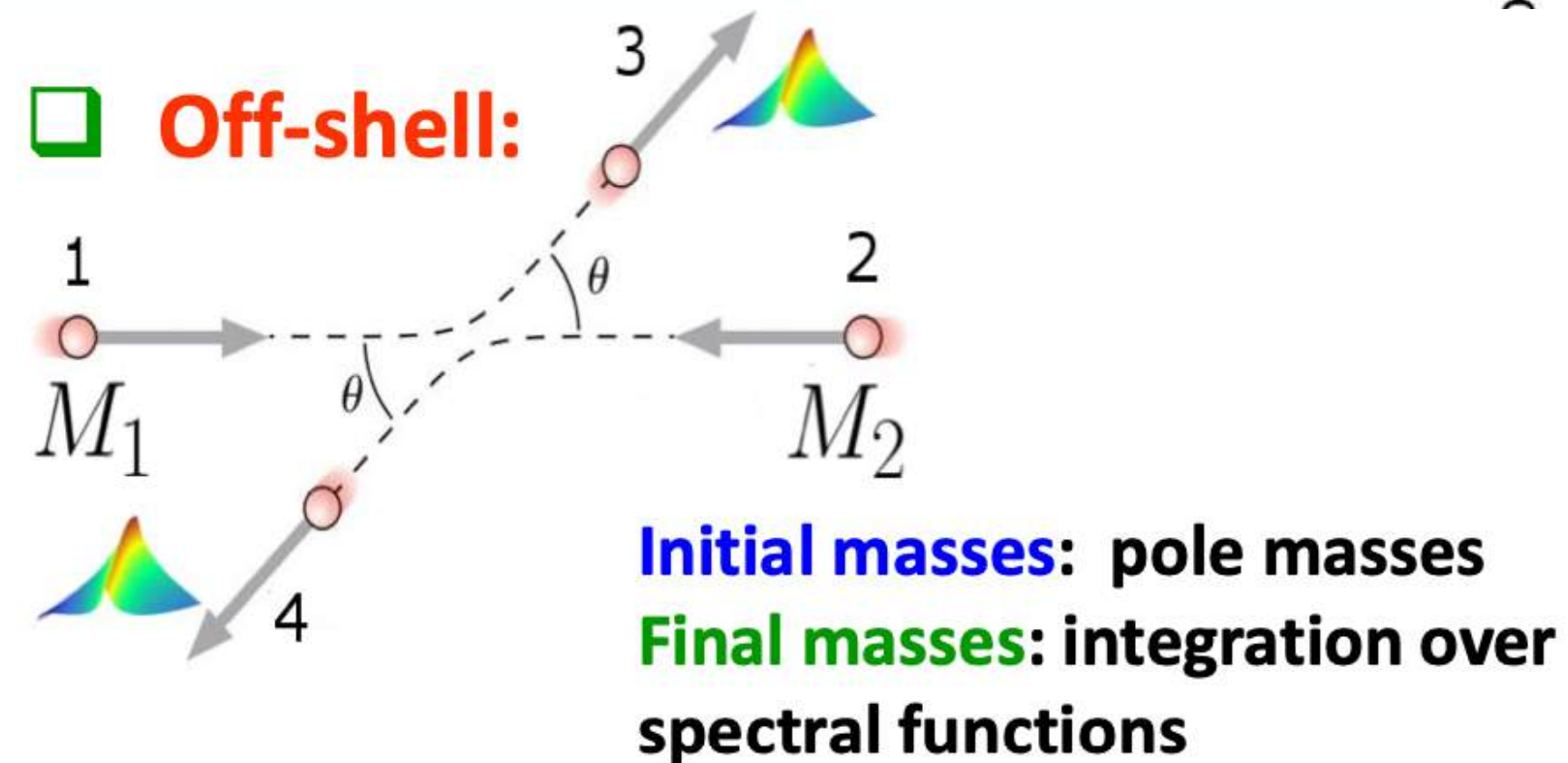
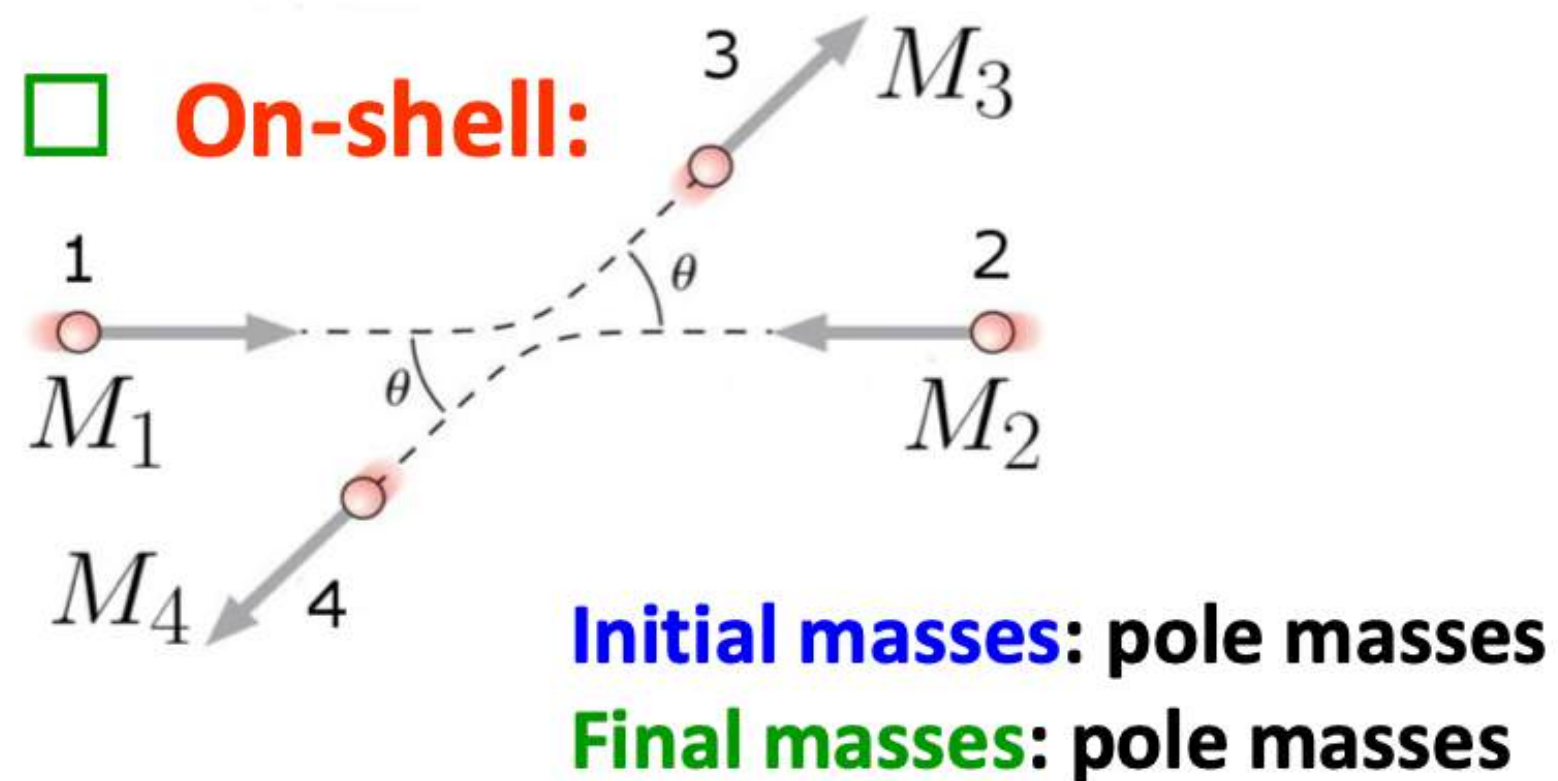
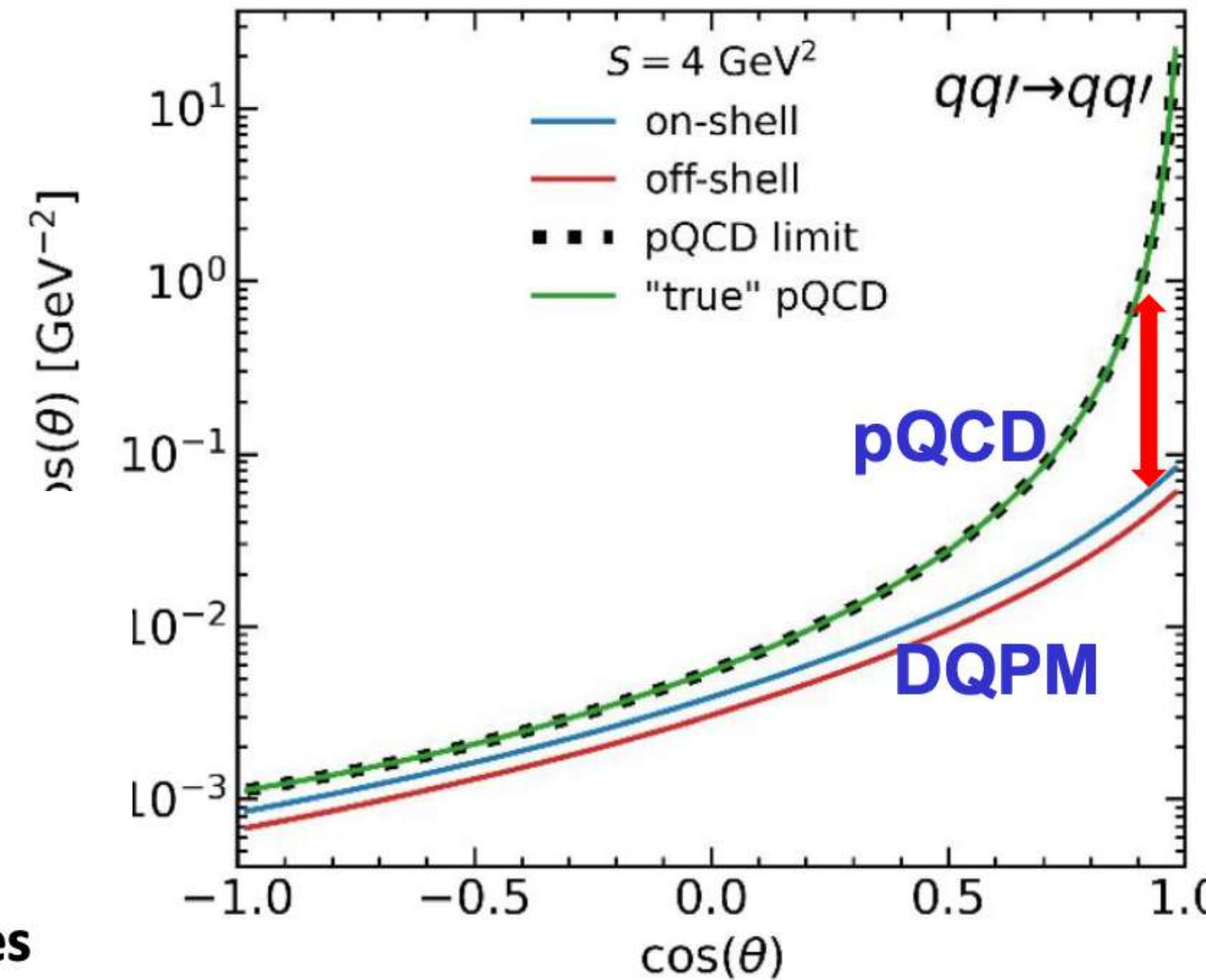
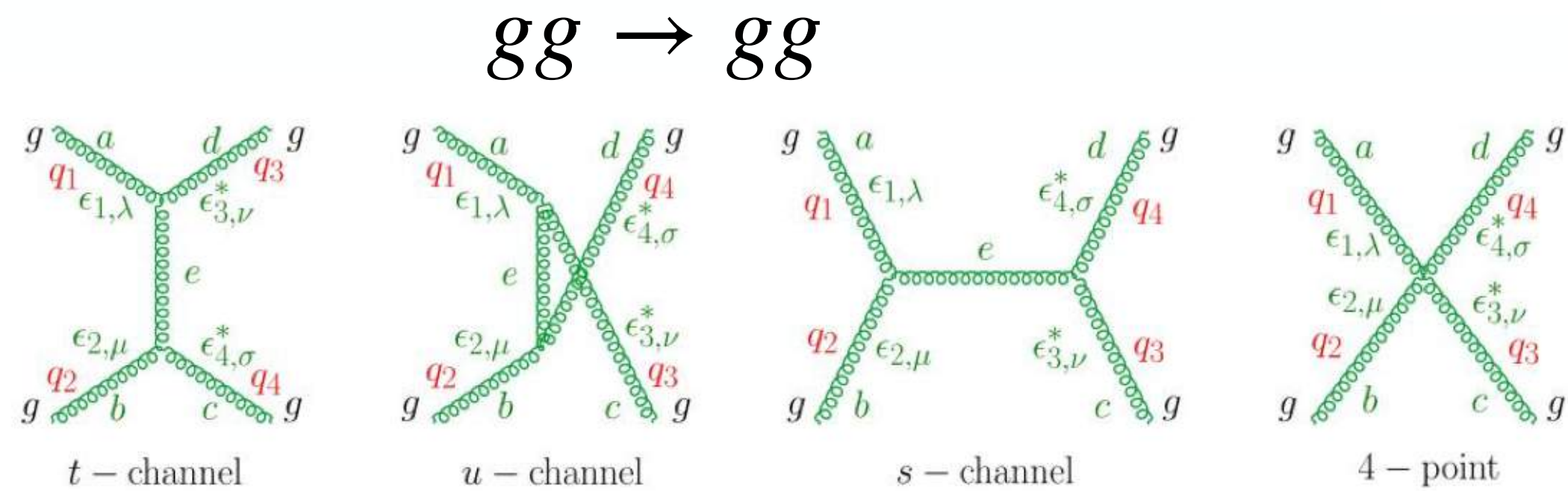
So far, only **leading order diagrams** included in PHSD

H. Berrehrah et al, PRC 93 (2016) 044914.

P. Moreau et al., PRC100 (2019) 014911.



$$\begin{aligned}
 \begin{array}{c} \mu, a \\ \text{---} \\ \nu, b \end{array} &= -i\delta_{ab} \frac{g^{\mu\nu} - q^\mu q^\nu / M_g^2}{q^2 - M_g^2 + 2i\gamma_g q_0} \\
 \begin{array}{c} i \\ \text{---} \\ j \end{array} &= i\delta_{ij} \frac{\not{q} + M_q}{q^2 - M_q^2 + 2i\gamma_q q_0}
 \end{aligned}$$



Off-shell parton hadronization

The hadronization occurs by quark-antiquark or 3 quark/3 antiquark **recombination** which is described by covariant transition rates \rightarrow energy-momentum conservation.

W. Cassing and E. Bratkovskaya, *PRC* 78 (2008) 034919.

$$\begin{aligned} \frac{dN_m(x, p)}{d^4x d^4p} &= Tr_q Tr_{\bar{q}} \delta^4(p - p_q - p_{\bar{q}}) \delta^4\left(\frac{x_q + x_{\bar{q}}}{2} - x\right) \\ &\times \omega_q \rho_q(p_q) \omega_{\bar{q}} \rho_{\bar{q}}(p_{\bar{q}}) |v_{q\bar{q}}|^2 W_m(x_q - x_{\bar{q}}, (p_q - p_{\bar{q}})/2) \\ &\times N_q(x_q, p_q) N_{\bar{q}}(x_{\bar{q}}, p_{\bar{q}}) \delta(\text{flavor, color}). \end{aligned}$$

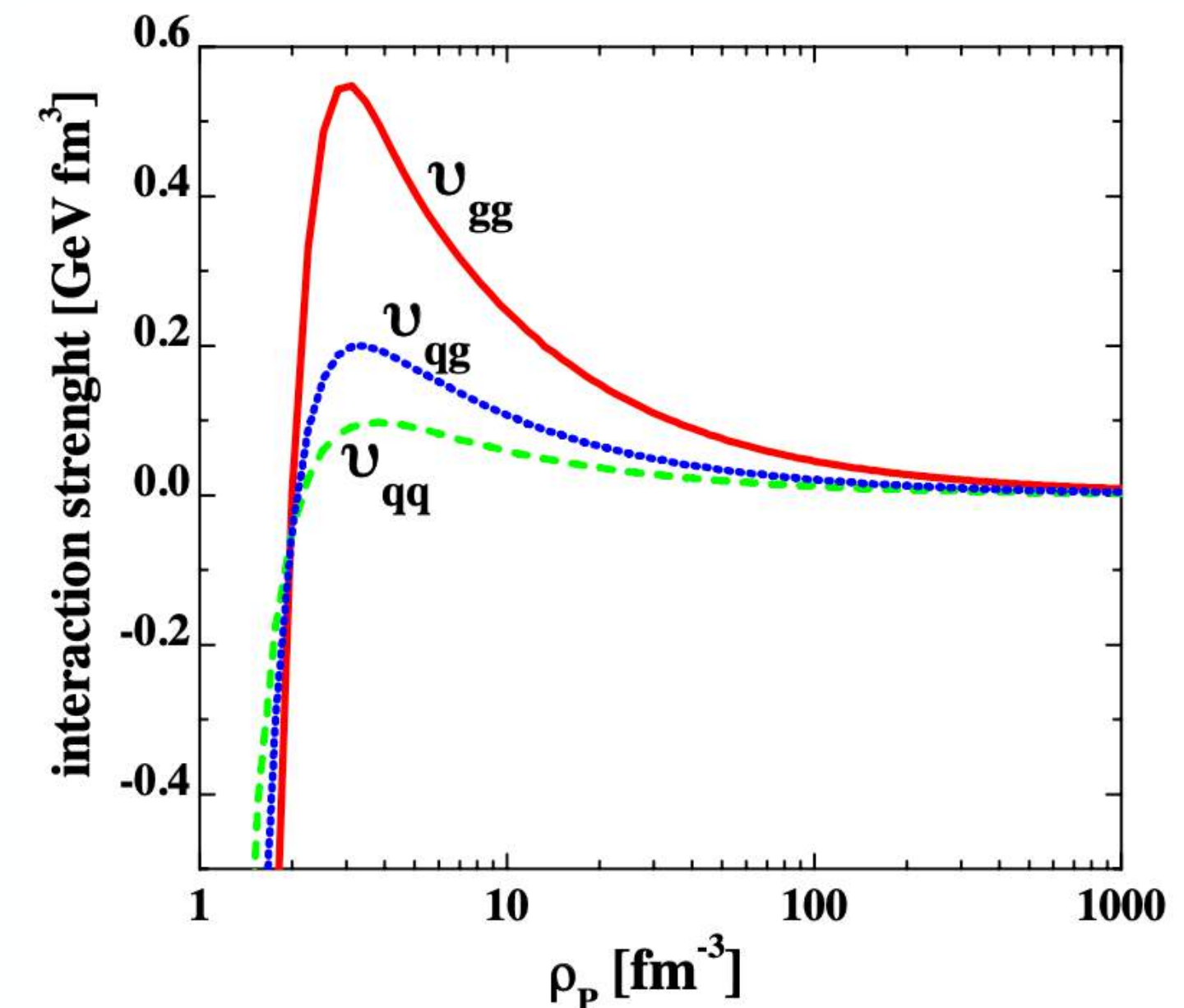
$$Tr_j = \sum_j \int d^4x_j \int \frac{d^4p_j}{(2\pi)^4} \quad \text{Sum over spin, flavor, color}$$

N_q : phase-space density

$v_{q\bar{q}}$: effective quark-antiquark interaction from the DQPM

$$W_m(\xi, p_\xi) = \exp\left(\frac{\xi^2}{2b^2}\right) \exp\left(2b^2(p_\xi^2 - (M_q - M_{\bar{q}})^2/4)\right)$$

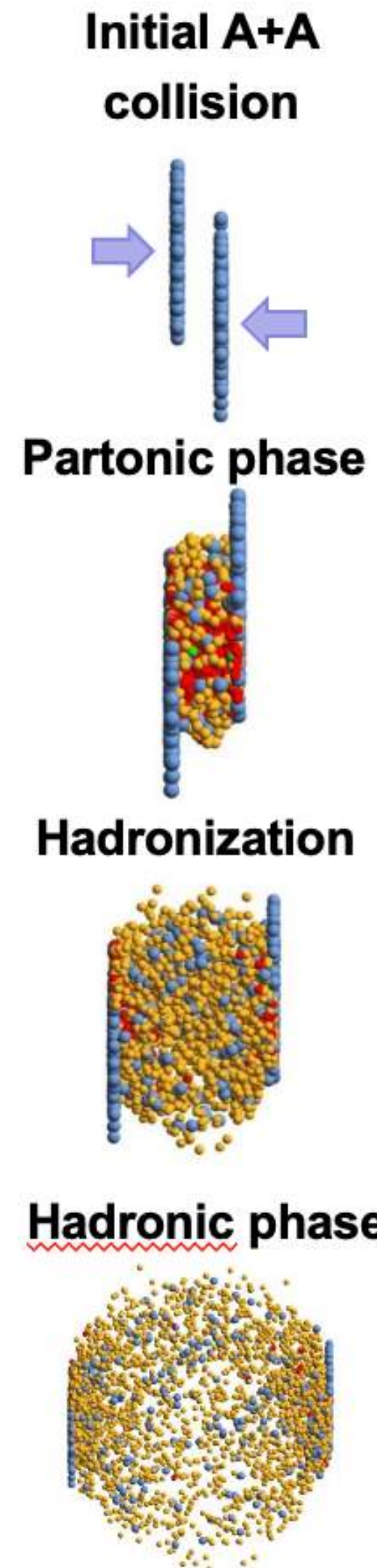
phase-space distribution of the formed “pre-hadron” (later decay to ground states)



Parton-Hadron-String-Dynamics (PHSD)

PHSD: based on the solution of **generalized off-shell transport equations** derived from Kadanoff-Baym many-body theory.

Website: <http://theory.gsi.de/~ebratkov/phsd-project/PHSD/index5.html>



□ **Initial A+A collisions :**
 $N+N \rightarrow$ **string formation** \rightarrow decay to pre-hadrons + leading hadrons

□ **Formation of QGP stage** if local $\varepsilon > \varepsilon_{\text{critical}}$:
 dissolution of **pre-hadrons** \rightarrow partons

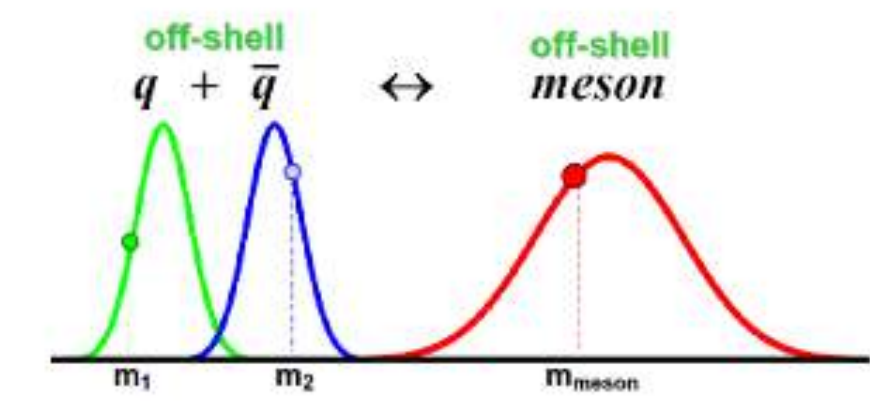
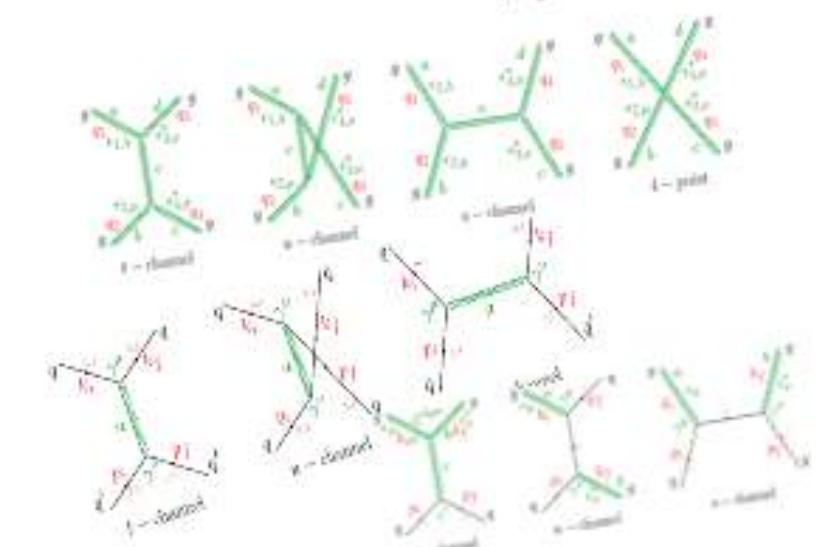
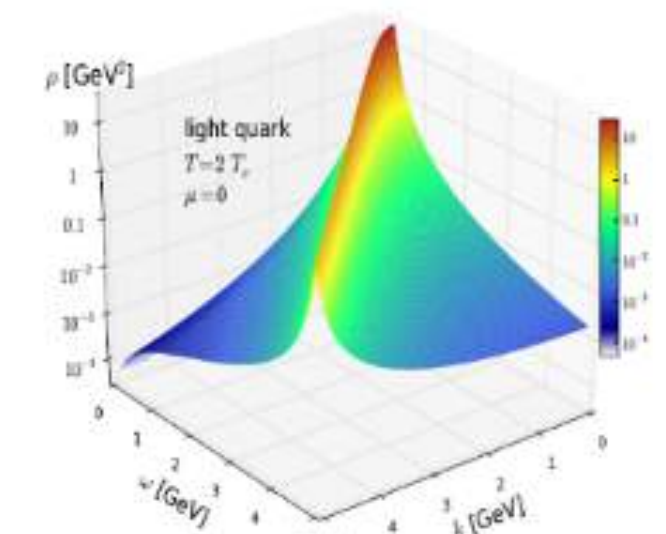
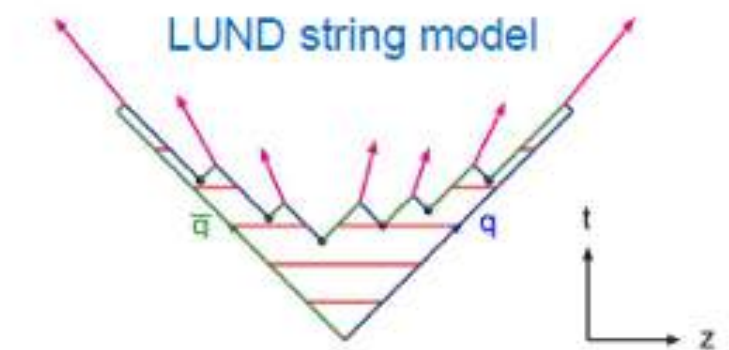
□ **Partonic phase - QGP:**
 QGP is described by the **Dynamical QuasiParticle Model (DQPM)**
 matched to reproduce **lattice QCD EoS** for finite T and μ_B (crossover)

- **Degrees-of-freedom:** strongly interacting quasiparticles:
massive quarks and gluons (g, q, q_{bar}) with sizeable collisional widths in a self-generated mean-field potential

- **Interactions:** (quasi-)elastic and inelastic collisions of partons

□ **Hadronization** to colorless **off-shell mesons and baryons:**
 Strict 4-momentum and quantum number conservation

□ **Hadronic phase:** hadron-hadron interactions – **off-shell HSD**
 including $n \leftrightarrow m$ selected reactions (for strangeness, anti-baryons, deuteron production)

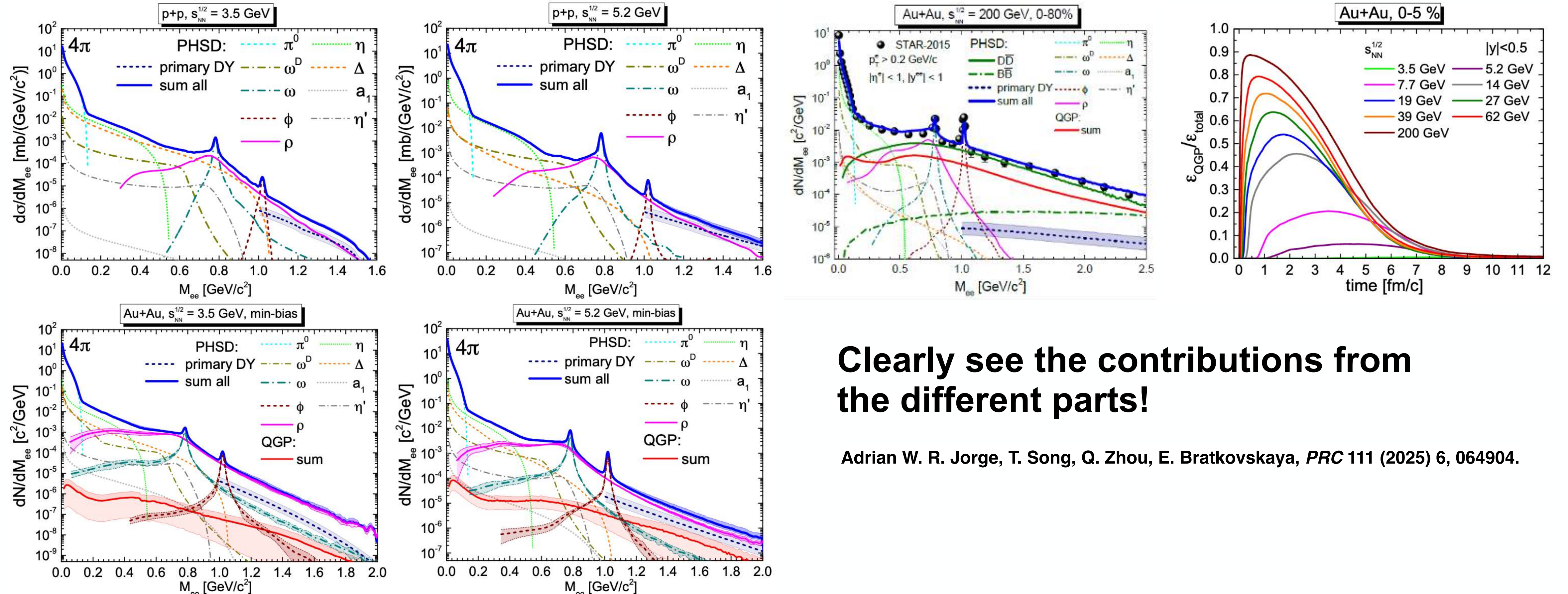


Highlights

□ **PHSD** provides a **good description of bulk observables** (y -, p_T -distributions, flow coefficients v_n , ...) from SIS to LHC

W. Cassing, E. Bratkovskaya, Phys.Rept. 308 (1999) 65;
E. Bratkovskaya, J. Aichelin, M. Thomere, S. Vogel, M. Bleicher, PRC 87 (2013) 064907; T. Song et al., Phys. Rev. C97 (2018), 064907; PRC 103, 044901 (2021); A. Palmese et al., PRC94 (2016) 044912;...

Recent: EM probe \rightarrow dilepton spectra including a **collisional broadening** of the **vector meson** spectral functions + primary DY + **QGP** + **correlated charm**



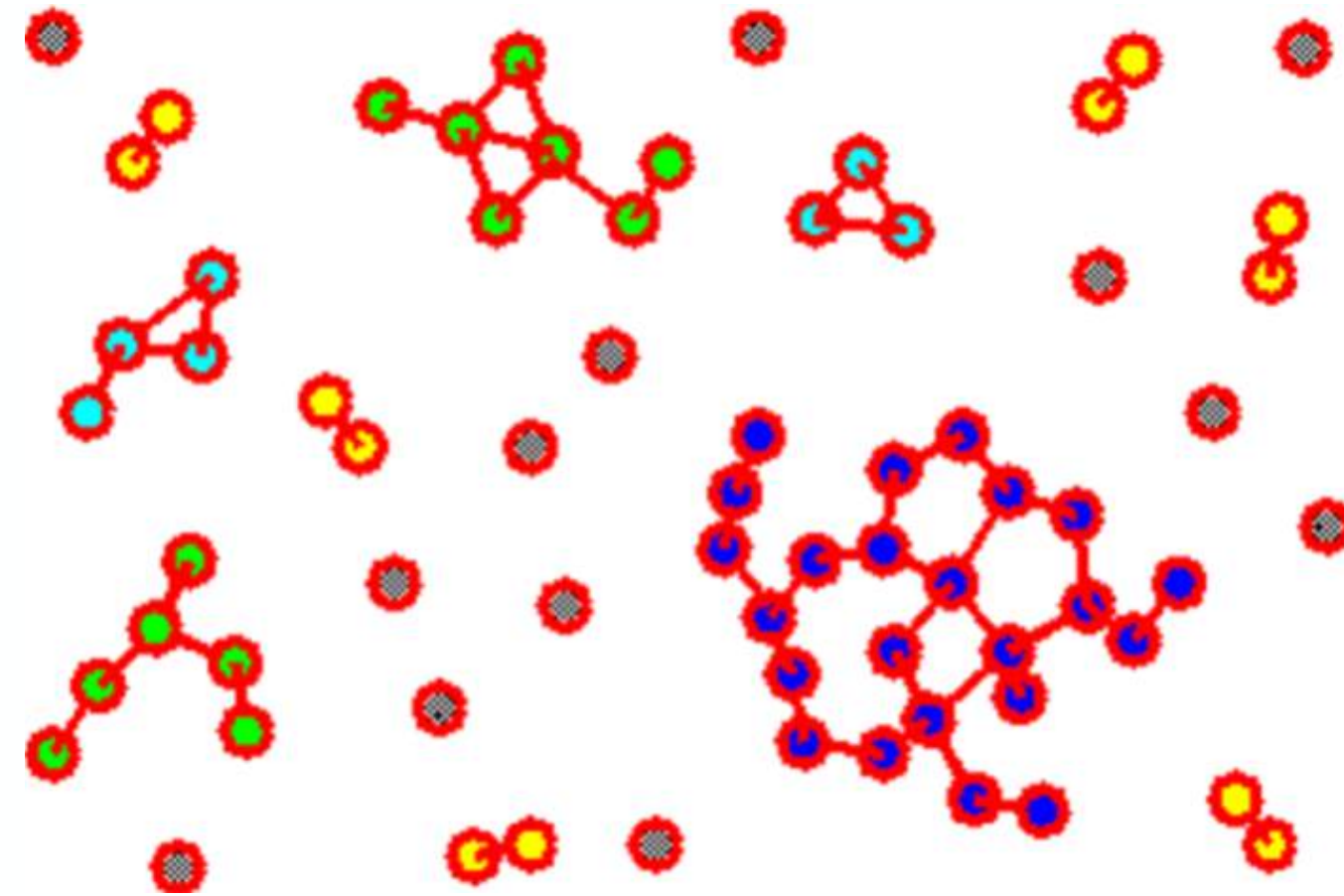
Clearly see the contributions from the different parts!

Adrian W. R. Jorge, T. Song, Q. Zhou, E. Bratkovskaya, PRC 111 (2025) 6, 064904.

PHQMD:



I. QMD for nucleons evolutions



BUU (Boltzmann–Uehling–Uhlenbeck) vs. QMD (Quantum Molecular Dynamics)

TMEP collaboration, PPNP, 125 (2022)103962.

Why QMD?

- “Bulk” observables for hadrons are rather similar in QMD and MF!
- Cluster formation is sensitive to nucleon dynamics:
 - **QMD** – allows to keep over time NN correlations by potential interaction
 - **MF** – correlations are smeared out
 - **Cascade** – no correlations by potential interactions

QMD (AMD, IQMD, ImQMD, LQMD, JAM,) is a n-body model but is limited to energies < 1.5 AGeV \rightarrow describes fragments at SIS energies

UrQMD is a n-body model but makes clusterization via coalescence and a statistical fragmentation model

PHQMD is a new n-body model not limited to low beam energies \rightarrow includes the formation of a quark gluon plasma at higher energies, at low energies similar to QMD



E. Bratkovskaya & J. Aichelin

PHQMD: a unified n-body microscopic transport approach for the description of heavy-ion collisions and **dynamical cluster formation** from low to ultra-relativistic energies

Realization: combined model **PHQMD = (PHSD & QMD) & (MST/SACA)**

J. Aichelin et al., PRC 101 (2020) 044905;
S. Gläsel et al., PRC 105 (2022) 1;
G. Coci et al., PRC 108 (2023) 1, 014902

Parton-Hadron-Quantum-Molecular Dynamics

Initialization → propagation of baryons:
QMD (Quantum-Molecular Dynamics)

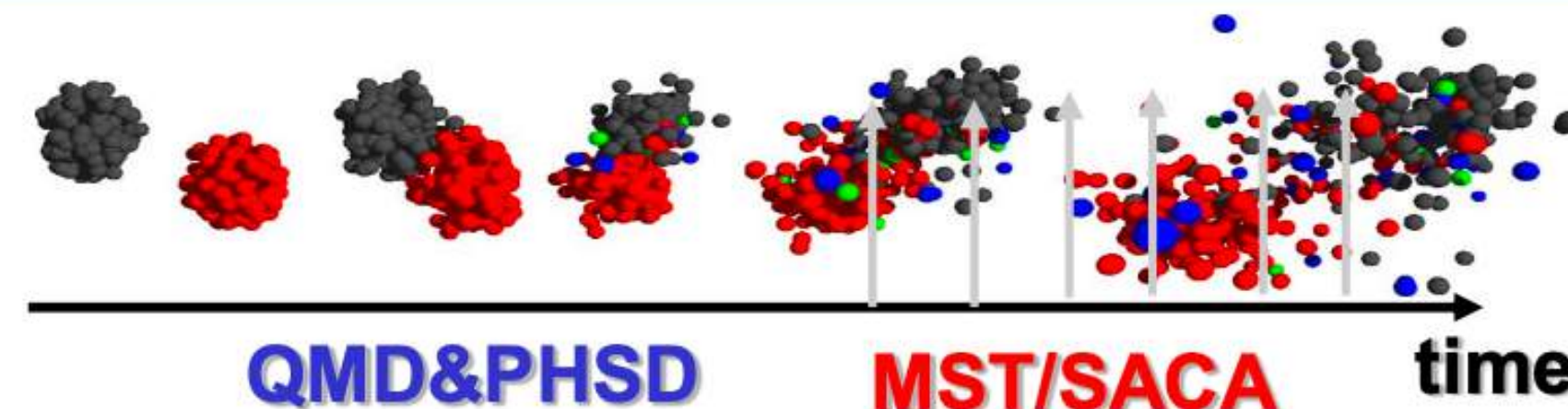
J.Aichelin, Phys. Rept. 202 (1991)

Propagation of partons (quarks, gluons) and mesons
+ **collision integral** = interactions of hadrons and partons (QGP)
from **PHSD (Parton-Hadron-String Dynamics)**

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215

Cluster recognition:
SACA (Simulated Annealing Clusterization Algorithm)
or **MST (Minimum Spanning Tree)**

R. Puri, J. Aichelin, J.Comp. Phys. 162 (2000) 245-260



PHQMD: EoM

Generalized Ritz variational principle: $\delta \int_{t_1}^{t_2} dt \langle \psi(t) | i \frac{d}{dt} - H | \psi(t) \rangle = 0.$

Many-body Hamiltonian: $H = \sum_i H_i = \sum_i (T_i + V_i) = \sum_i (T_i + \sum_{j \neq i} V_{i,j})$

Many-body wave function: $\psi(t) = \prod_{i=1}^N \psi(\mathbf{r}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t)$

Ansatz: trial wave function for one particle “i” : **Gaussian** with width L centered at “ r_{i0} ”, “ p_{i0} ”

$$\psi(\mathbf{r}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t) = C e^{-\frac{1}{4L} \left(\mathbf{r}_i - \mathbf{r}_{i0}(t) - \frac{\mathbf{p}_{i0}(t)}{m} t \right)^2} \cdot e^{i \mathbf{p}_{i0}(t) (\mathbf{r}_i - \mathbf{r}_{i0}(t))} \cdot e^{-i \frac{\mathbf{p}_{i0}^2(t)}{2m} t}$$

Equations-of-motion (EoM) in coordinate and momentum space:

J.Aichelin, Phys. Rept. 202 (1991)

$$\dot{r}_{i0} = \frac{\partial \langle H \rangle}{\partial p_{i0}} \quad \dot{p}_{i0} = -\frac{\partial \langle H \rangle}{\partial r_{i0}}$$

PHQMD: Potential

2-body potential:

$$\begin{aligned}
 V_{ij} &= V(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_{i0}, \mathbf{r}_{j0}, \mathbf{p}_{i0}, \mathbf{p}_{j0}, t) \\
 &= V_{\text{Skyrme loc}} + V_{\text{mom}} + V_{\text{Coul}} \\
 &= \frac{1}{2} t_1 \delta(\mathbf{r}_i - \mathbf{r}_j) + \frac{1}{\gamma + 1} t_2 \delta(\mathbf{r}_i - \mathbf{r}_j) \rho^{\gamma-1}(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_{i0}, \mathbf{r}_{j0}, t) \quad \text{Skyrme} \\
 &+ \boxed{V(\mathbf{r}_i, \mathbf{r}_j, \mathbf{p}_{i0}, \mathbf{p}_{j0})} + \frac{1}{2} \frac{Z_i Z_j e^2}{|\mathbf{r}_i - \mathbf{r}_j|}, \\
 &\quad \text{momentum dependent} \qquad \qquad \qquad \text{Coulomb}
 \end{aligned}$$

In infinite matter a potential corresponds to the EoS:

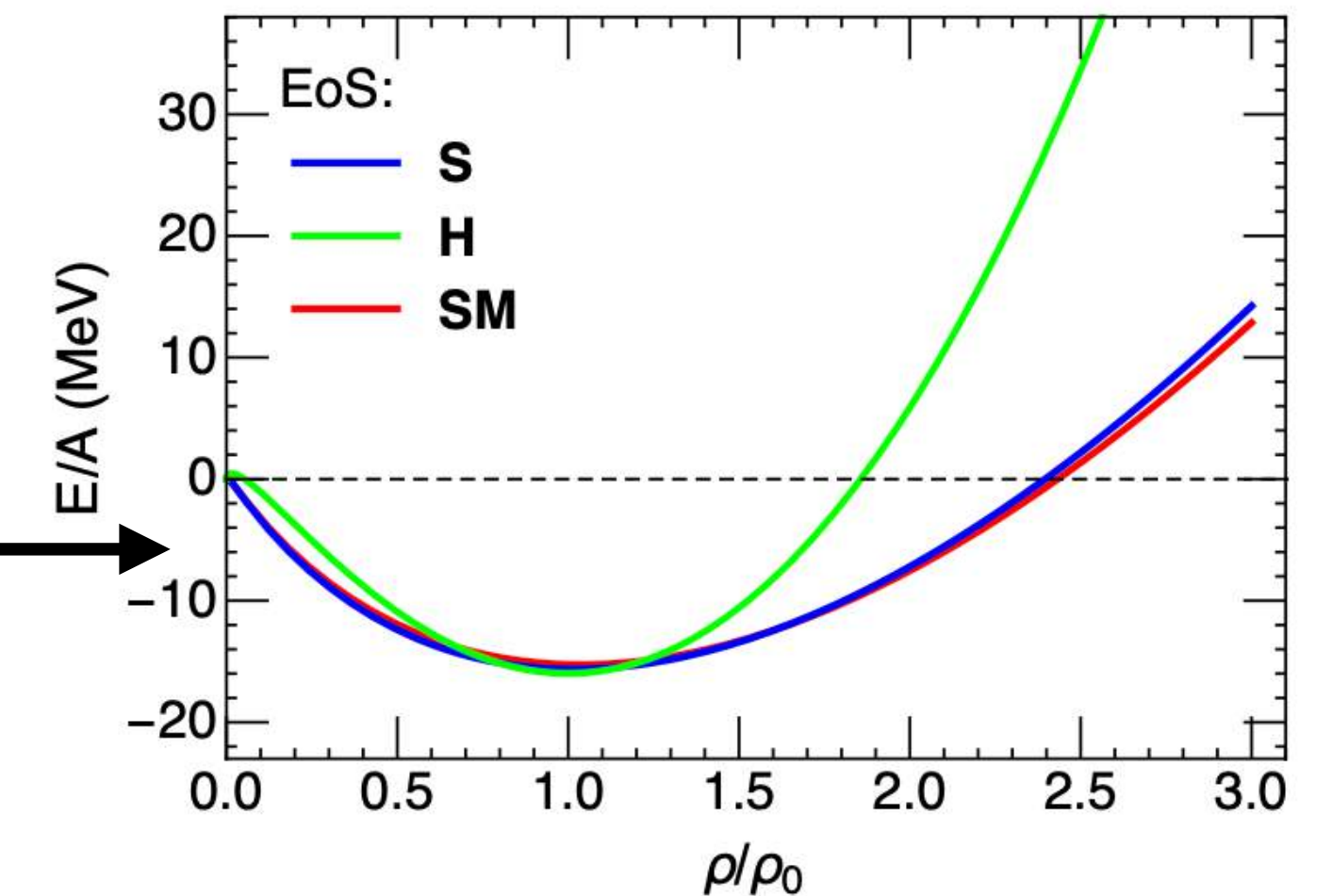
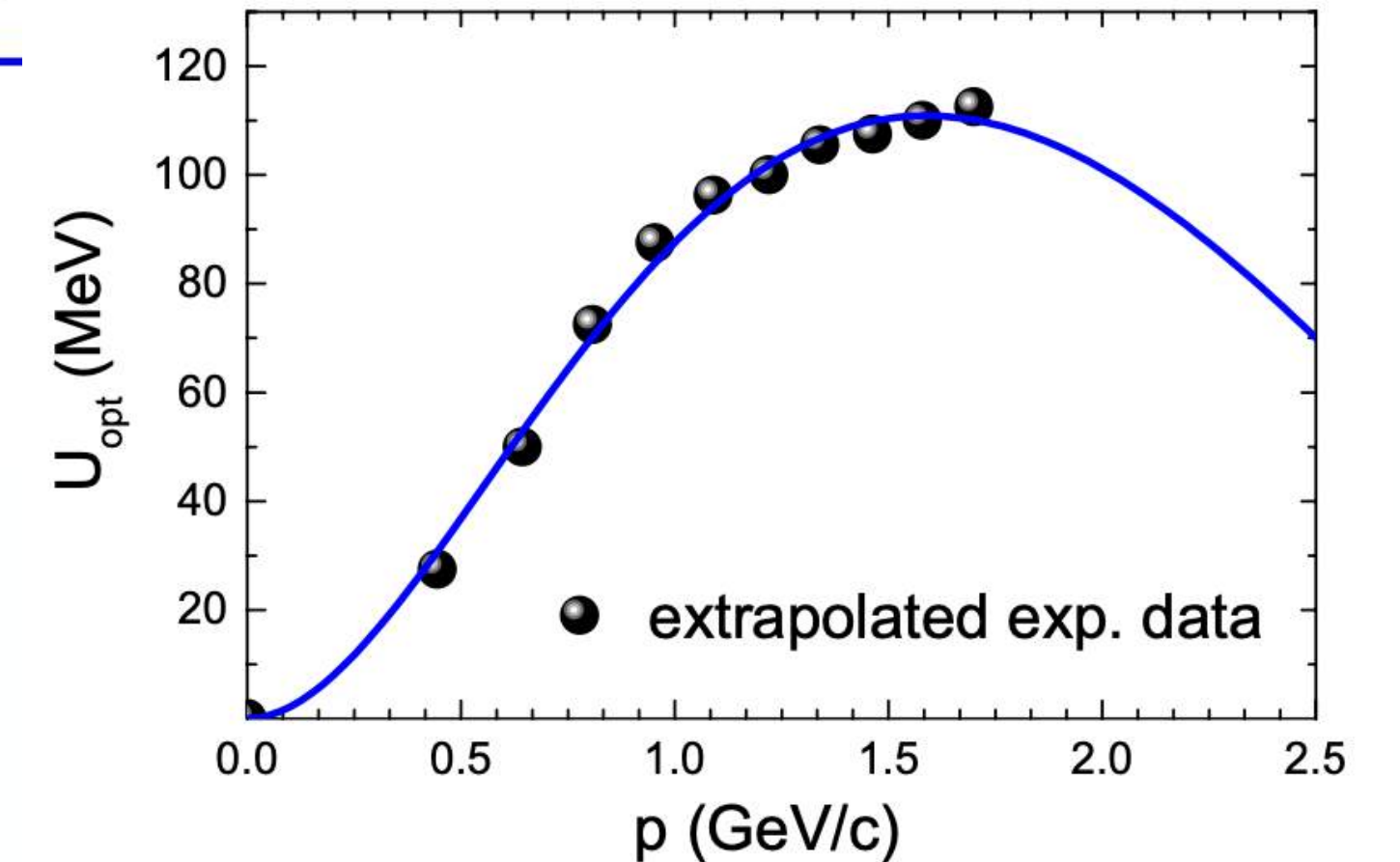
$$E/A(\rho) = \frac{3}{5} E_F + V_{\text{Skyrme stat}}(\rho) + V_{\text{mom}}(\rho)$$

$$V_{\text{Skyrme stat}} = \alpha \frac{\rho}{\rho_0} + \beta \left(\frac{\rho}{\rho_0} \right)^\gamma, \quad V_{\text{mom}} = (a \Delta p + b \Delta p^2) \exp(-c \sqrt{\Delta p}) \frac{\rho}{\rho_0}$$

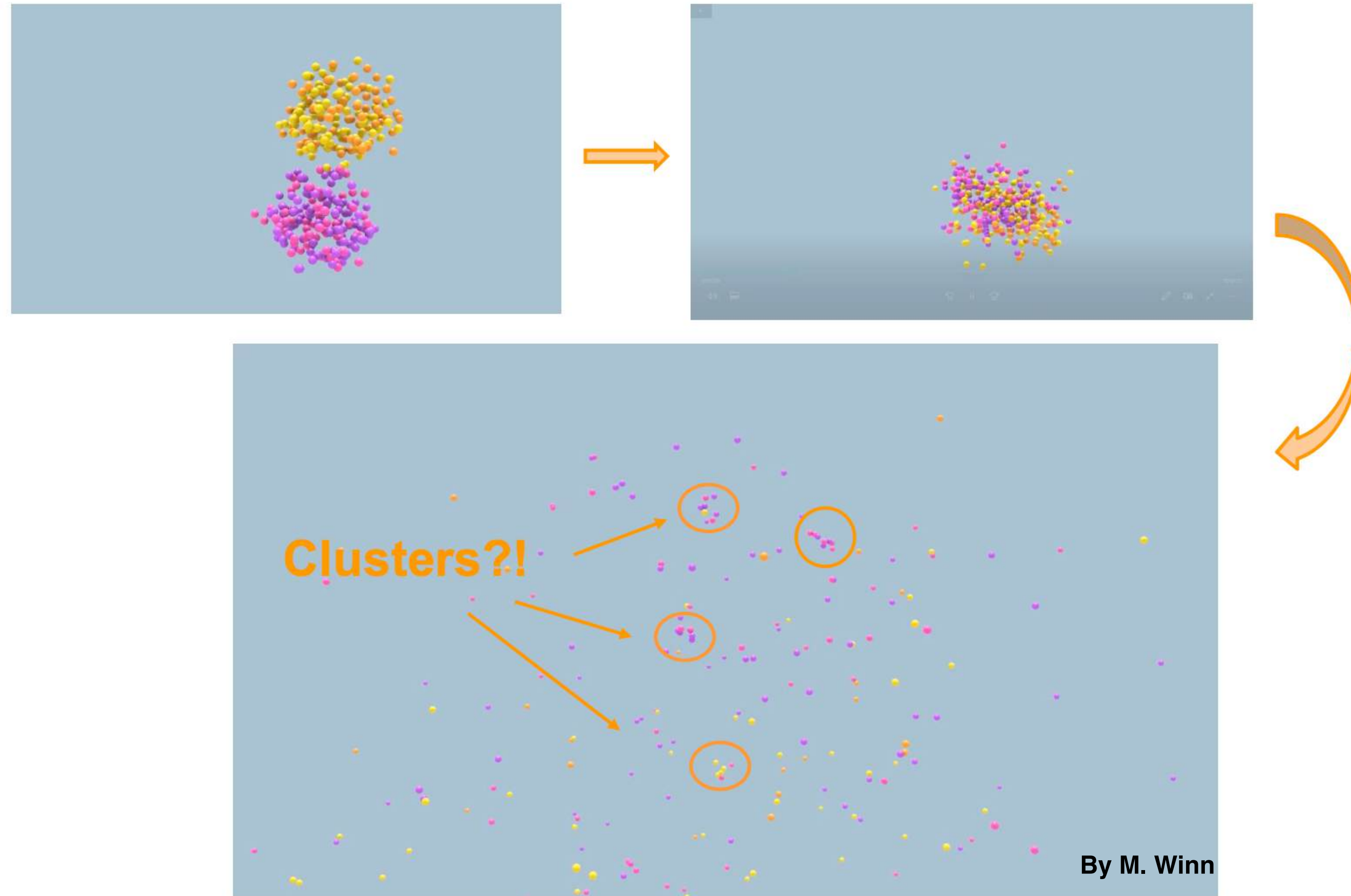
Compression modulus
K of nuclear matter:

$$K = -V \frac{dP}{dV} = 9\rho^2 \frac{\partial^2 (E/A(\rho))}{(\partial \rho)^2} \Big|_{\rho=\rho_0}$$

E.o.S.	α [MeV]	β [MeV]	γ	K [MeV]
S	-383.5	329.5	1.15	200
H	-125.3	71.0	2.0	380
SM	-478.87	413.76	1.10	200
	a [MeV ⁻¹]	b [MeV ⁻²]	c [MeV ⁻¹]	
	236.326	-20.73	0.901	



PHQMD: Cluster formation

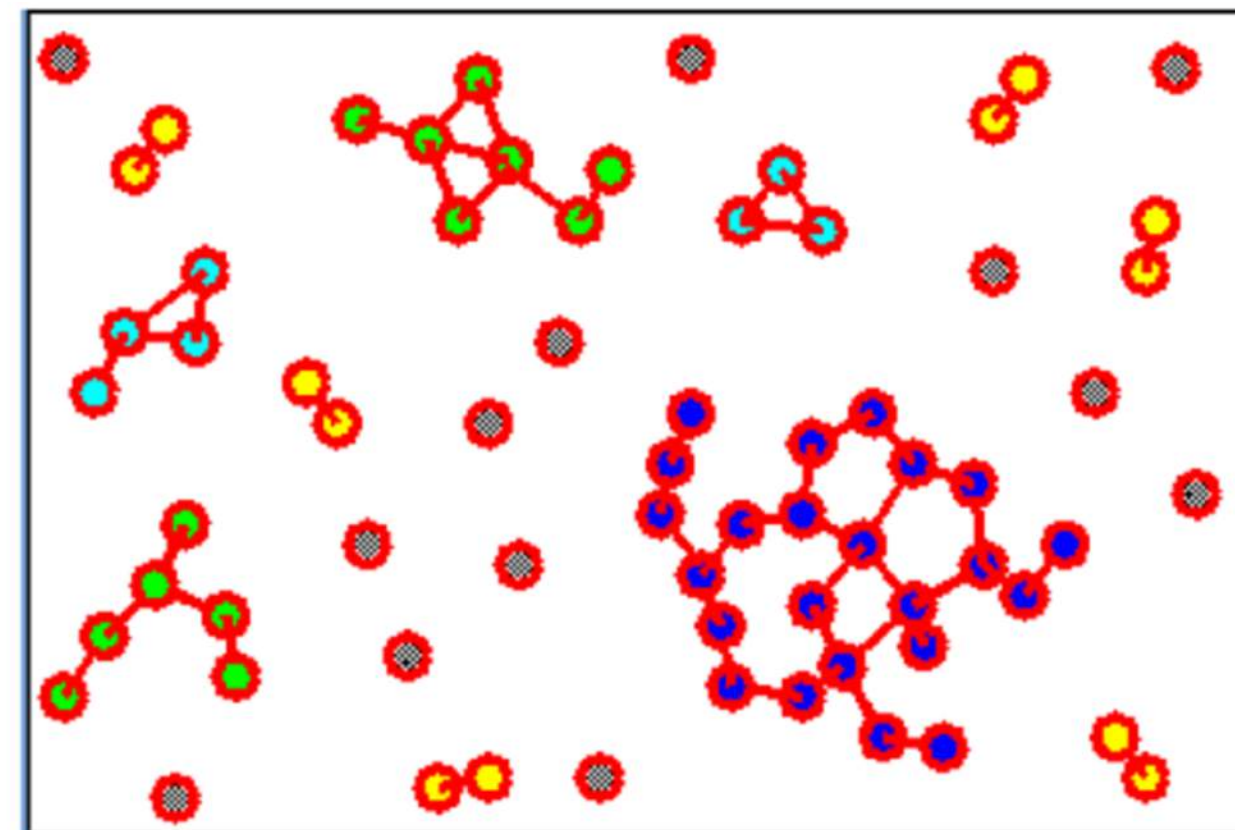


The **Minimum Spanning Tree (MST)** is a **cluster recognition** method applicable for the (asymptotic) **final states** where coordinate space correlations may only survive for bound states.

1. Two particles are 'bound' if their distance in the cluster rest frame fulfills:

$$|\vec{r}_i - \vec{r}_j| \leq 4 \text{ fm}$$

2. Particle is bound to a cluster if it binds with at least one particle of the cluster



Advanced MST G. Coci et al., Phys.Rev.C 108 (2023) 014902

3. Negative binding energy for identified clusters
4. **Stabilization procedure** – recombine the final “lost” nucleons back into clusters if they left the cluster without rescattering

PHQMD: Kinetic

1. hadronic inelastic reactions $NN \leftrightarrow d\pi$, $\pi NN \leftrightarrow d\pi$, $NNN \leftrightarrow dN$

2. hadronic elastic $\pi + d$, $N + d$ reactions

SMASH: D. Oliinychenko et al., PRC 99 (2019) 044907;
 J. Staudenmaier et al., PRC 104 (2021) 034908
 AMPT: R.Q. Wang et al. PRC 108 (2023) 3;
 PHQMD: G. Coci et al., PRC 108 (2023) 014902.

$$P_{3,2}(\sqrt{s}) = F_{spin} F_{iso} P_{2,3}(\sqrt{s}) \frac{E_1^f E_2^f}{2E_3 E_4 E_5} \frac{R_2(\sqrt{s}, m_1, m_2)}{R_3(\sqrt{s}, m_3, m_4, m_5)} \frac{1}{\Delta V_{cell}}$$

Energy and momentum
of final particles

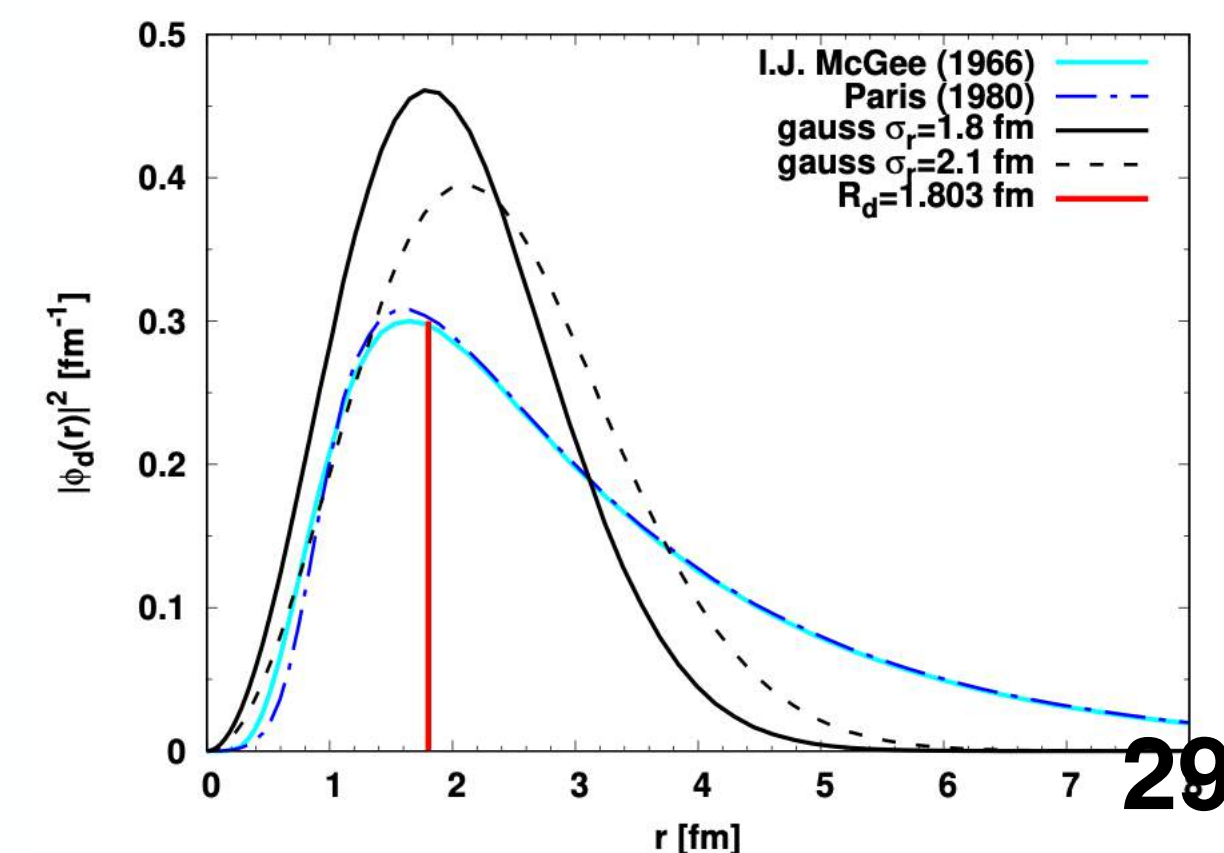
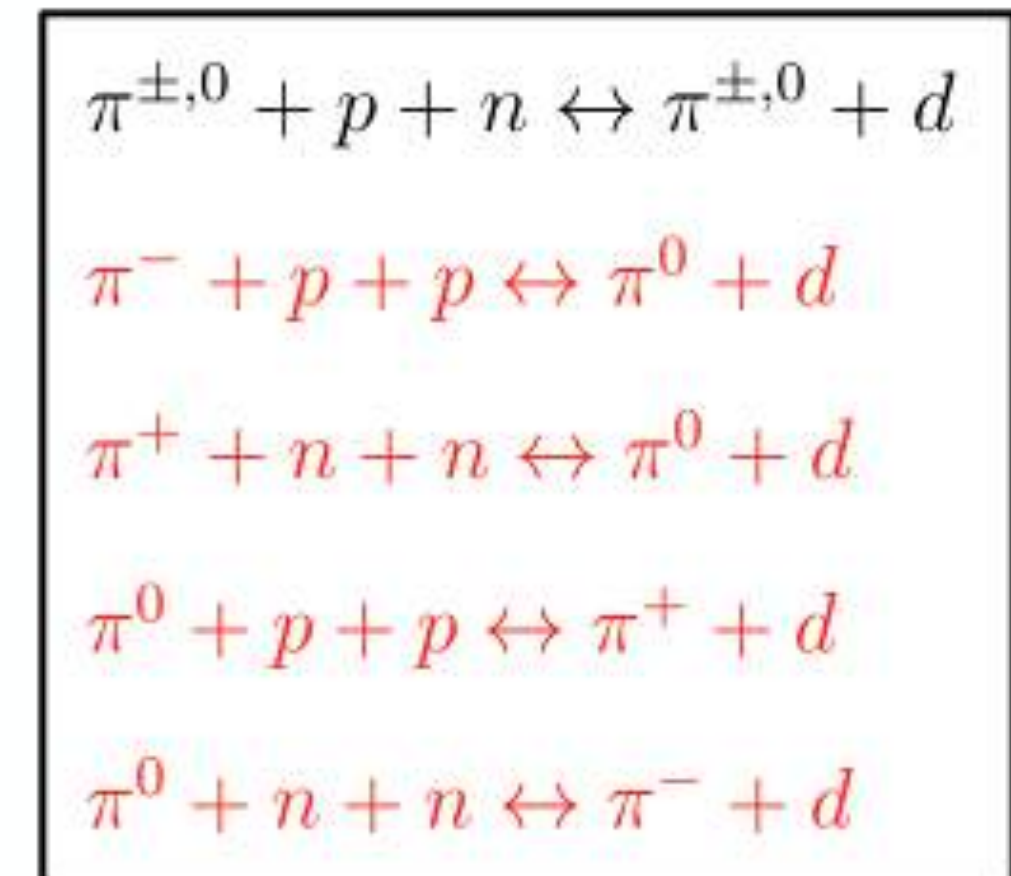
2,3-body phase space
integrals
[Byckling, Kajantie]

+ inclusion of all possible isospin channels (enhance d production)
 + accounting of quantum properties of d (suppress d production) :

1) the **finite-size of d in coordinate space** (d is not a point-like particle)
 assume that a d can not be formed in a high density region

Excluded-Volume Condition: $|\vec{r}(i)^* - \vec{r}(d)^*| < R_d$

2) the **momentum correlations of p and n inside d**
 project the relative momentum of $p + n$ pair on d wave-function in
 momentum space $|\phi_d(\mathbf{p})|^2$ which lead to a strong reduction of d production.



PHQMD: Coalescence

Clusters formation at a **freeze-out time** by coalescence radii in coordinate and momentum space
 coalescence parameters from UrQMD → in PHQMD: $\Delta P < 0.285$ GeV and $\Delta R < 3.575$ fm

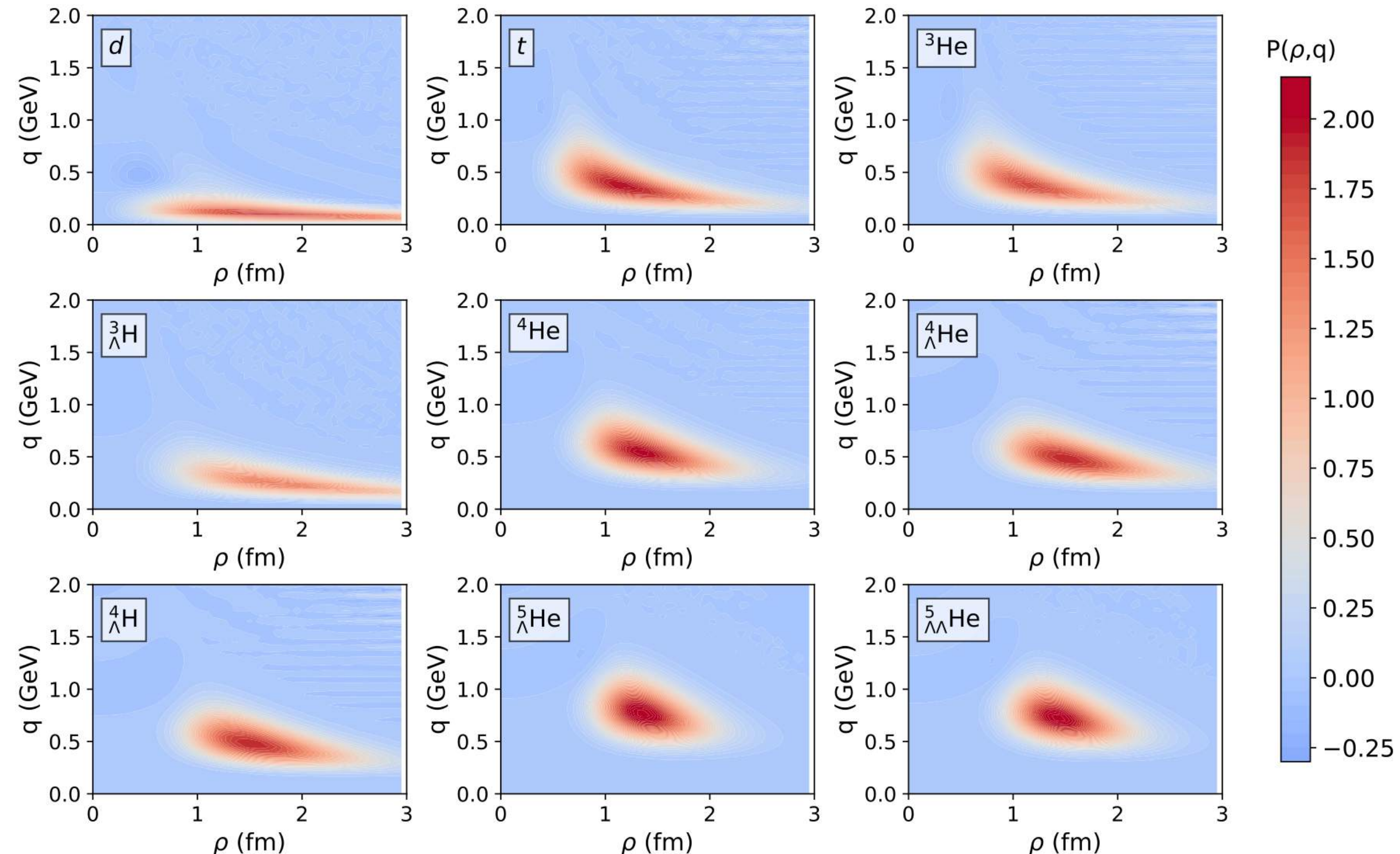
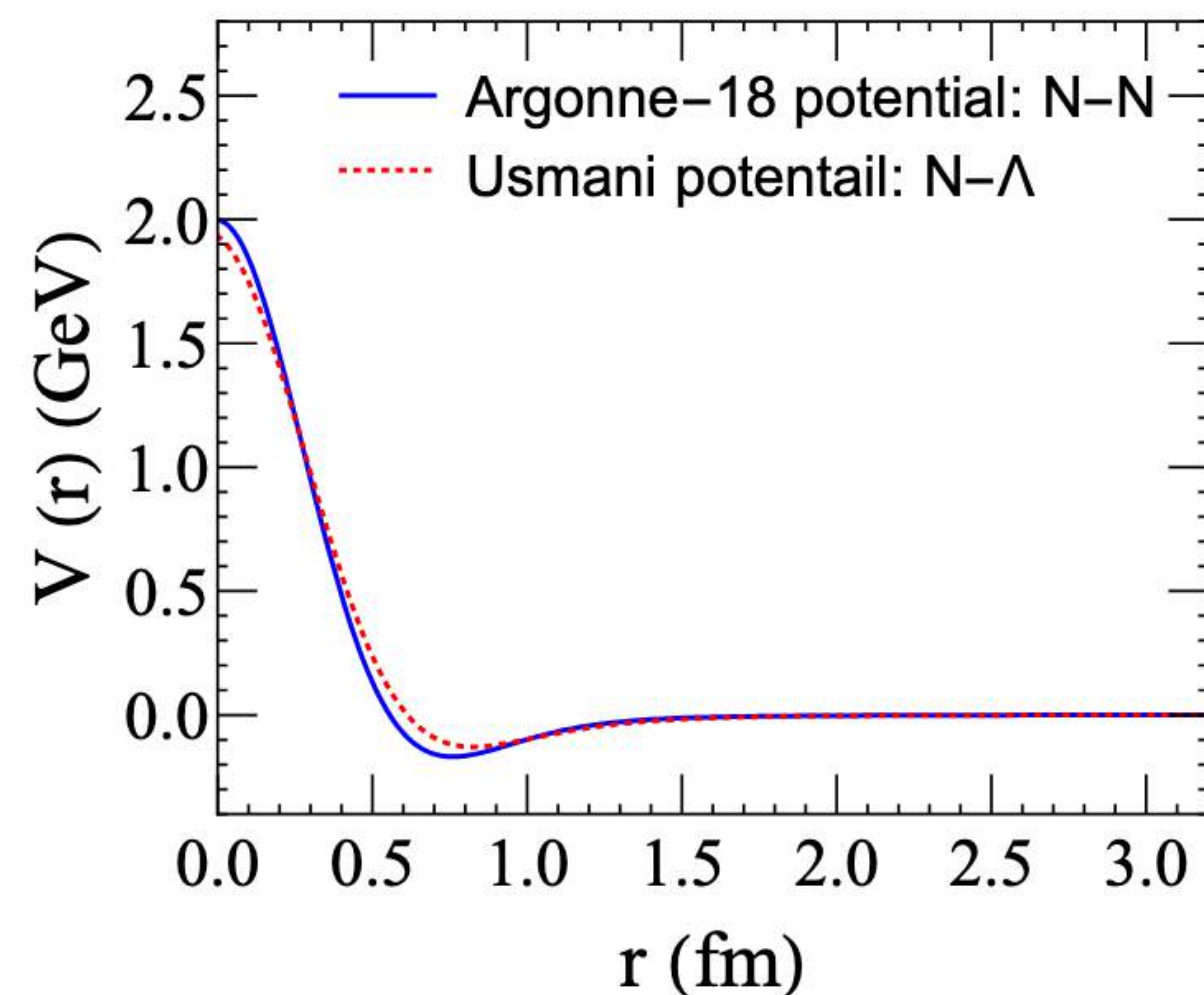
Recent: More realistic Wigner function from N-body Schrödinger equations →

J. Zhao, J. Aichelin, E. Bratkovskaya, arXiv: 2508.05814

$$H = \sum_{i=1}^N \frac{\mathbf{p}_i^2}{2m_i} + \sum_{i<j} V_{ij}.$$

$$\mathbf{R} = \frac{1}{M} \sum_{i=1}^N m_i \mathbf{r}_i$$

$$\mathbf{x}_j = \sqrt{\frac{M_j m_{j+1}}{M_{j+1} \mu}} \left(\mathbf{r}_{j+1} - \frac{1}{M_j} \sum_{i=1}^j m_i \mathbf{r}_i \right)$$

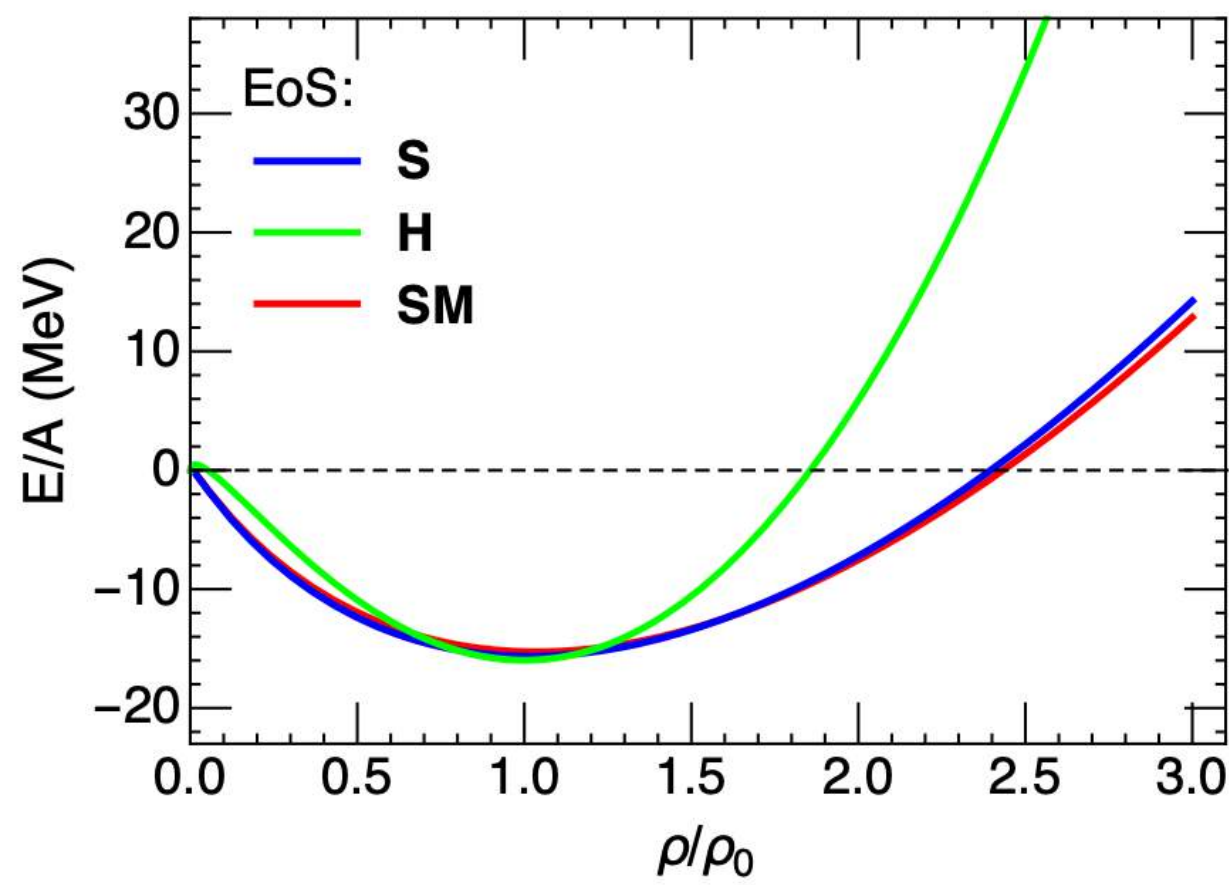
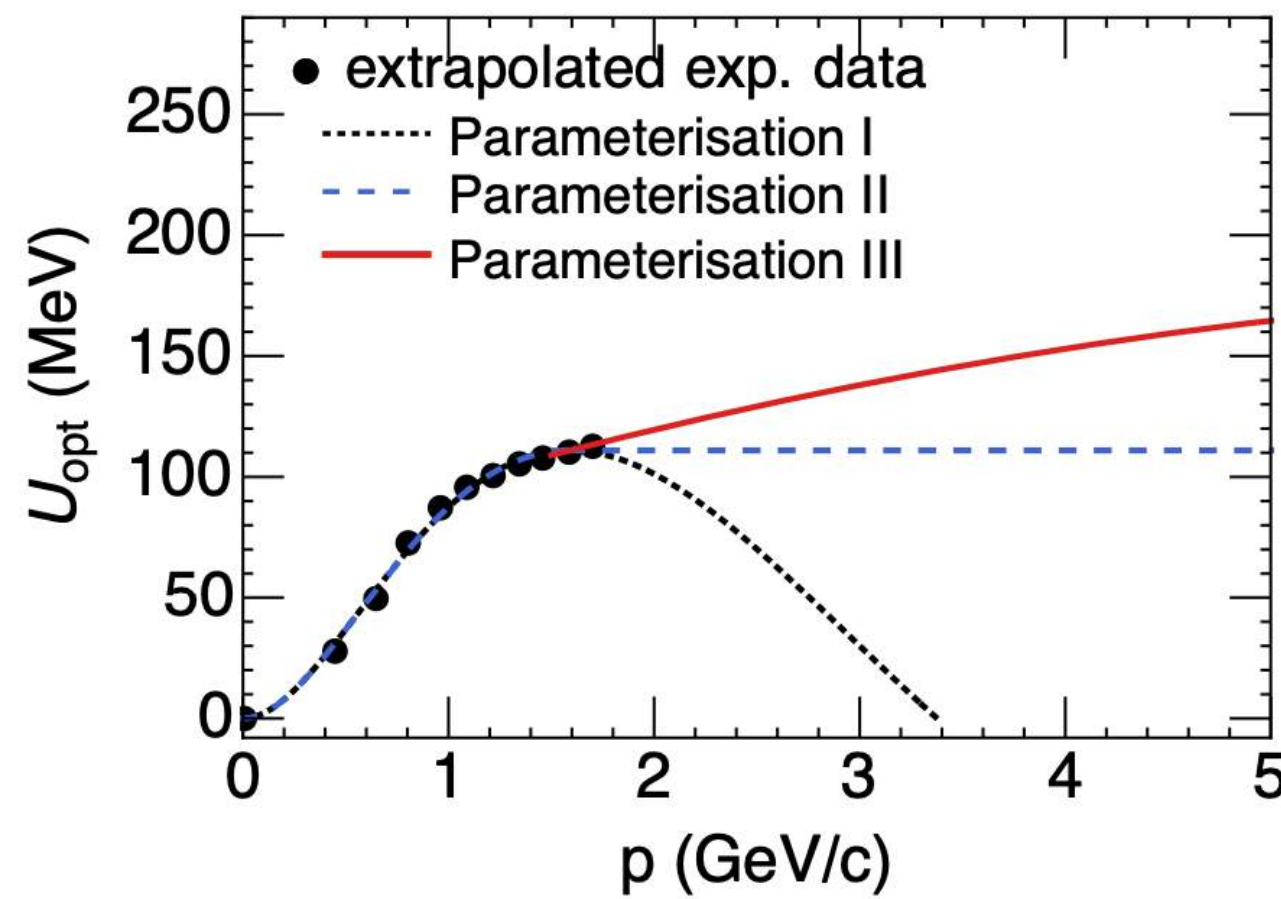


Highlights

□ **PHQMD** provides a **good description of cluster and hypernuclei observables** (y -, p_T -distributions, flow coefficients v_n , ...)

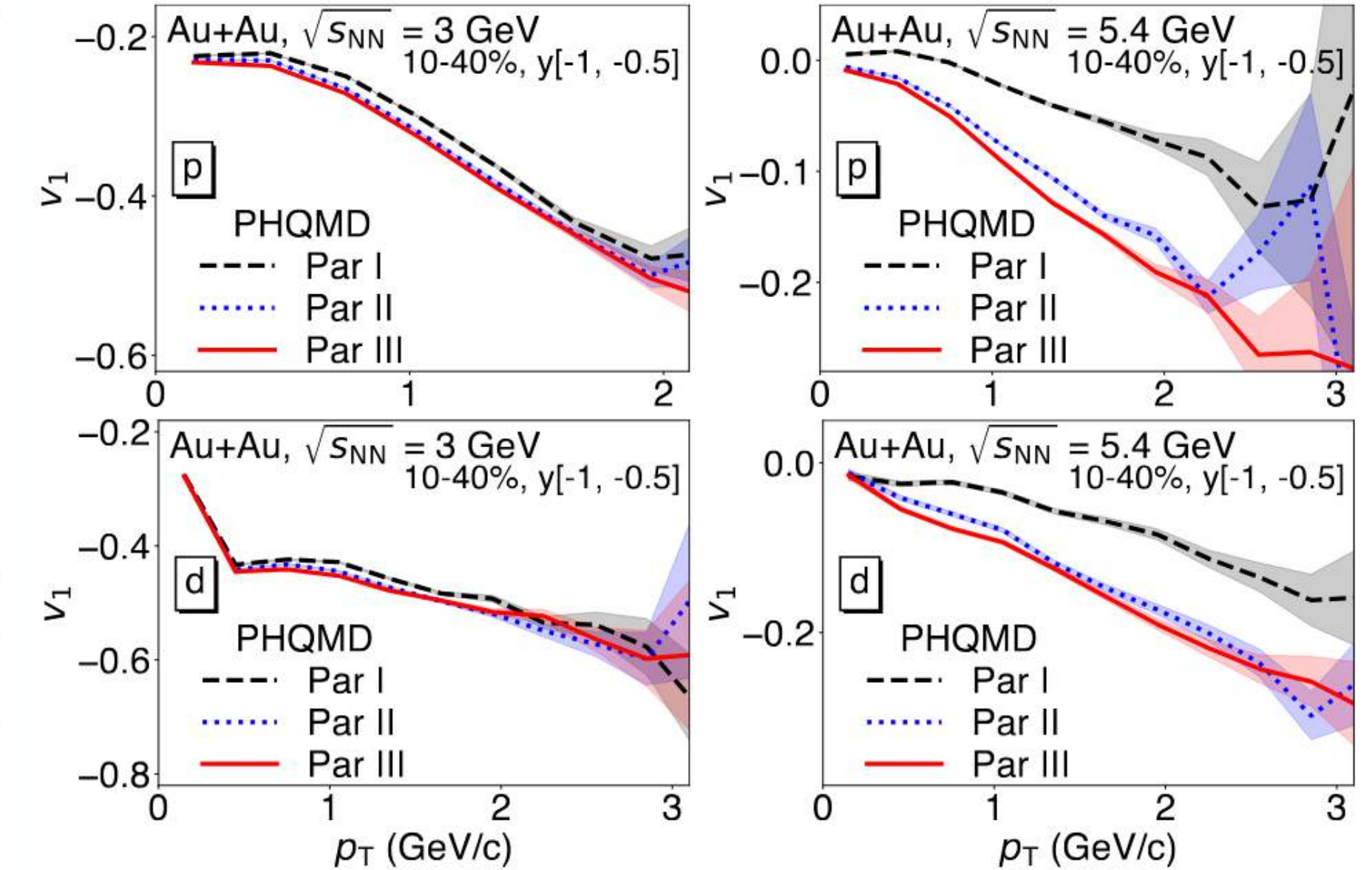
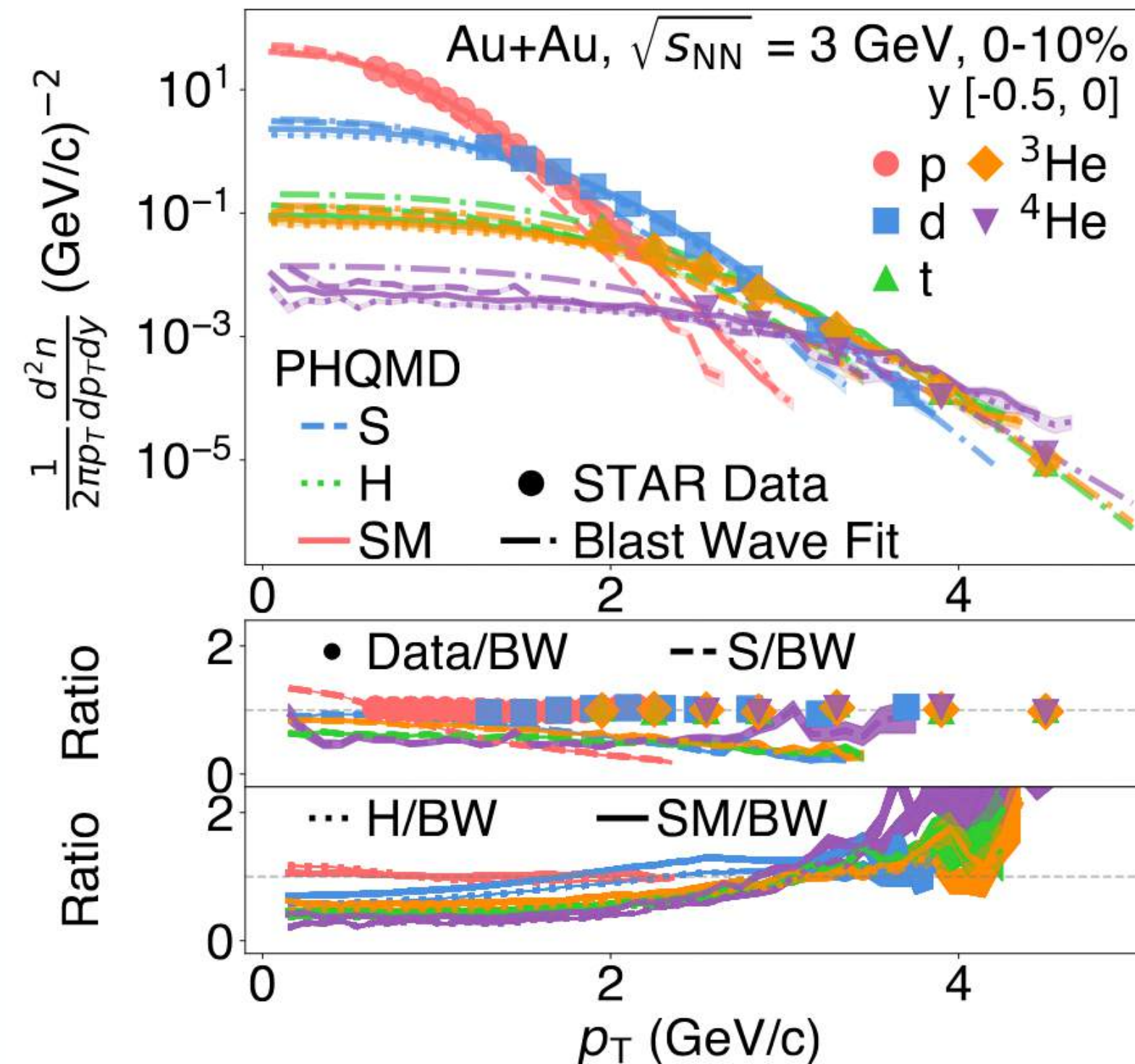
S. Gläsel et al, *Phys.Rev.C* 105 (2022) 1, 014908. G. Coci et al., *PRC* 108 (2023) 014902. V. Kireyeu et al., arXiv:2411.04969; ...

Recent: probe EoS with clusters and hypernuclei—> support **SM**, need more data at higher energy (4-7GeV) to probe the **mom-dependent potential**.



Probing of EoS with clusters and hypernuclei arXiv: 2507.14255.

Yingjie Zhou¹, Iouri Vassiliev¹, Yue-Hang Leung², Norbert Herrmann^{2,1}, Susanne Gläsel³, Viktor Kireyeu⁴, Michael Winn⁵, Jörg Aichelin^{5,6}, Christoph Blume^{2,1,7}, Elena Bratkovskaya^{1,7,8}, Vadim Voronyuk⁴, Nu Xu^{9,10}, Jiaxing Zhao^{7,8}

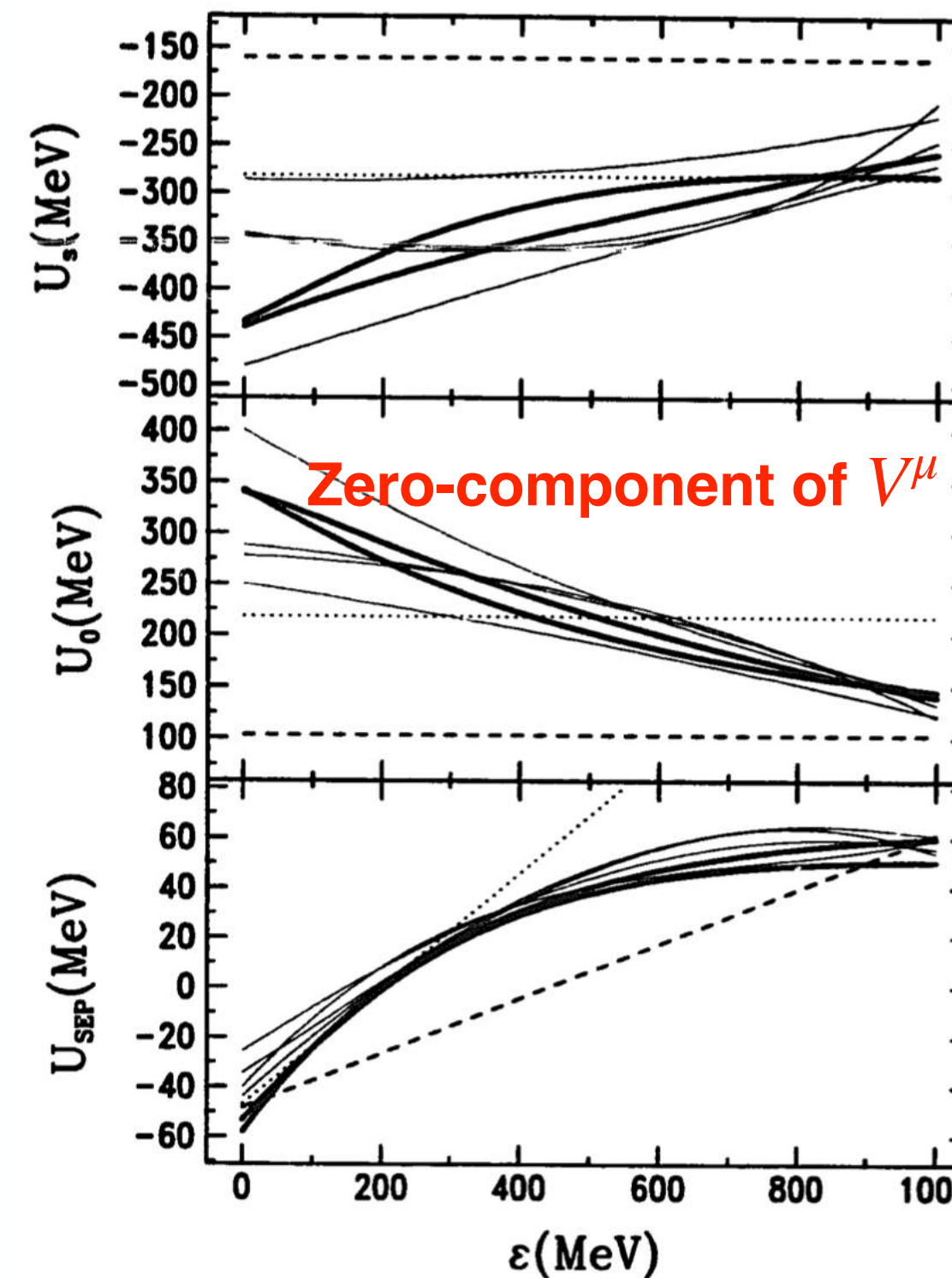
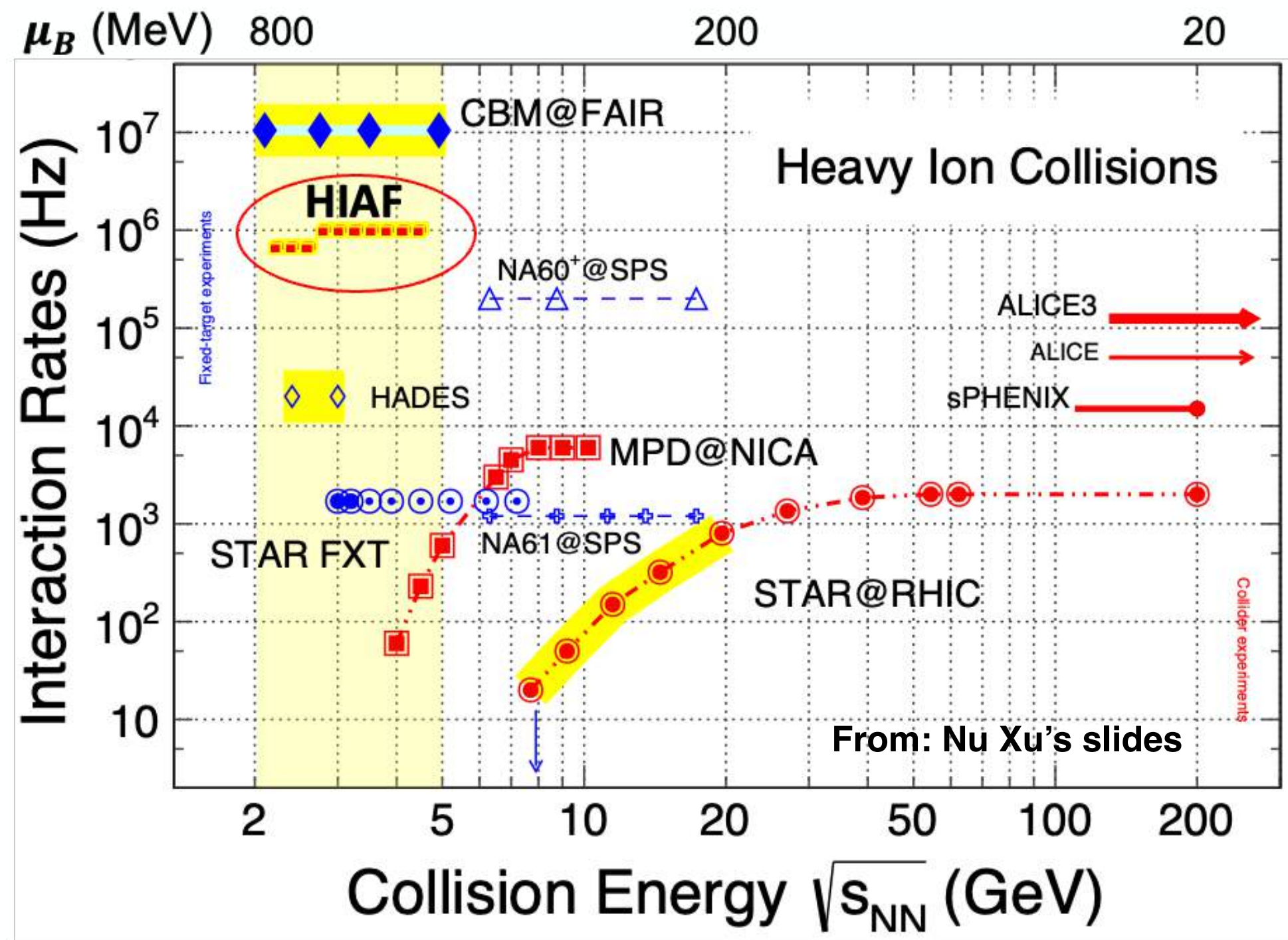


PHQMD:

II. Relativistic QMD for FAIR, NICA, HIAF energies

Relativistic version

- ➔ **Relativistic effects play a crucial role** —> a fully covariant N-body transport approach is needed for probing the EoS, cluster formation,...
- ➔ N-N interaction in the medium includes **a scalar potential U_s and a vector potential V^μ**



PHYSICAL REVIEW C

VOLUME 47, NUMBER 1

JANUARY 1993

Global Dirac phenomenology for proton-nucleus elastic scattering

E. D. Cooper, S. Hama, and B. C. Clark

Department of Physics, The Ohio State University, Columbus, Ohio 43210

R. L. Mercer

IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598

(Received 31 August 1992)

Energy-dependent global Dirac optical model potentials are found by fitting proton elastic scattering data in the energy range 20–1040 MeV for ^{12}C , ^{16}O , ^{40}Ca , ^{90}Zr , and ^{208}Pb . Three different energy- and atomic-mass-number-dependent global Dirac optical potentials are also obtained. A number of characteristic features of the potentials are discussed. In addition, the mean free path, the effective mass m_e^* , the Dirac mass M^* , and the relativistic energy shift E^* are calculated.

$$\{\alpha \cdot p + \beta(m + U_s^{\text{tot}}(r; \epsilon)) + U_0^{\text{tot}}(r; \epsilon)\} \Psi(r; \epsilon) = E \Psi(r; \epsilon)$$

$$\left\{ \frac{p^2}{2m} + U_{\text{SEP}}(r; \epsilon) + U_{\text{s.o.}}(r; \epsilon) \frac{\sigma \cdot L}{r} + C(r; \epsilon) \right\} \Psi_{\pm} = \frac{p_x^2}{2m} \Psi_{\pm}$$

$$U_{\text{SEP}} = U_s^{\text{tot}} + U_0^{\text{tot}} + \frac{1}{2m} (U_s^{\text{tot}^2} - U_0^{\text{tot}^2}) + \frac{U_0^{\text{tot}}}{m} \epsilon$$

Schrödinger equivalent potential (SEP)

Relativistic version

LORENTZ-COVARIANT DESCRIPTION OF INTERMEDIATE ENERGY HEAVY-ION REACTIONS IN RELATIVISTIC QUANTUM MOLECULAR DYNAMICS*

RQMD

Tomoyuki MARUYAMA, S.W. HUANG, N. OHTSUKA¹, Guoqiang LI² and Amand FAESSLER

Institut für Theoretische Physik der Universität Tübingen, D-7400 Tübingen, Germany

J. AICHELIN

Institut für Theoretische Physik der Universität Heidelberg, D-6900 Heidelberg, Germany

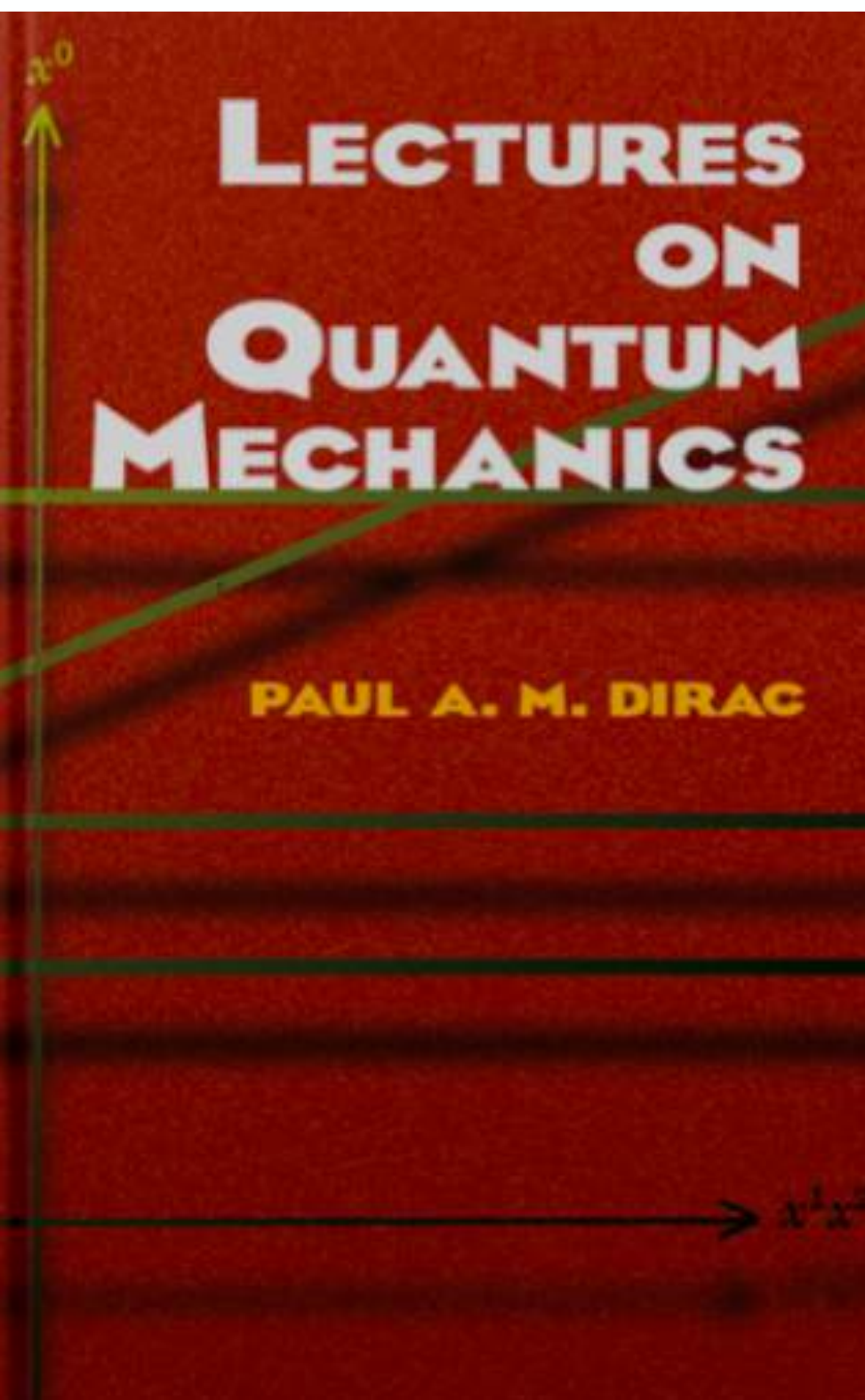
Received 1 February 1991
(Revised 3 May 1991)

Poincaré Invariant Hamiltonian Dynamics: Modelling Multi-hadronic Interactions in a Phase Space Approach

HEINZ SORGE, HORST STÖCKER, AND WALTER GREINER

*Institut für Theoretische Physik,
Johann Wolfgang Goethe-Universität
Frankfurt/Main, Germany*

Received January 9, 1989



Dirac constraint dynamics: For N-body system,
8N dof.

N first-class constraints (on-shell conditions), $H_i = p_{i,\mu} p_i^\mu - m_i$,

7N dof.

Additional N second-class constraints (the last constraint serve to
pick a clock for all particles)

6N+1 dof. Solvable!

RQMD, UrQMD only with scalar potential (Skyrme-type potentials)

JAM with both the scalar and vector potential (RMF)

Relativistic QMD

On-shell conditions

$$H_1 = p_{1,\mu}^* p_1^{*\mu} - m_1^2 + \Phi_1,$$

$$H_2 = p_{2,\mu}^* p_2^{*\mu} - m_2^2 + \Phi_2.$$

$$p_{i,\mu}^* \equiv p_{i,\mu} - A_{i,\mu}$$

$$\Phi_1 = \Phi_2 = \Phi(\sqrt{-q_T^2})$$

$$q_T^\mu = \left(g^{\mu\nu} - \frac{P^\mu P^\nu}{P^2} \right) q_\nu = q^\mu - \frac{q_\nu P^\nu}{P^2} P^\mu$$

Second-class constraints

$$\chi_1 = \frac{1}{2}(q_1^\mu - q_2^\mu)U_\mu = 0, \quad \chi_2 = \frac{1}{2}(q_1^\mu + q_2^\mu)U_\mu - \tau = 0, \quad \text{Poincaré invariant (world lines condition)}$$

Equations of motion(Poisson brackets)

$$\frac{dq_i^\mu}{d\tau} = \{q_i^\mu, H\} = \lambda_1 \{q_i^\mu, H_1\} + \lambda_2 \{q_i^\mu, H_2\},$$

$$\frac{dp_i^\mu}{d\tau} = \{p_i^\mu, H\} = \lambda_1 \{p_i^\mu, H_1\} + \lambda_2 \{p_i^\mu, H_2\}.$$

J. Zhao, J. Aichelin, E. Bratkovskaya, arXiv: 2511.13975

$$\frac{dq_i^\mu}{d\tau} = \sum_{k=1}^2 \lambda_k \left(2p_k^{*\nu} \frac{\partial p_{k,\nu}^*}{\partial p_{i,\mu}^*} \right),$$

$$\frac{dp_i^\mu}{d\tau} = - \sum_{k=1}^2 \lambda_k \left(2p_k^{*\nu} \frac{\partial p_{k,\nu}^*}{\partial q_{i,\mu}^*} + \frac{\partial \Phi_k}{\partial q_{i,\mu}^*} \right).$$

Non-relativistic limit is fulfilled

Only scalar potential

Neglects the momentum-dependent derivatives & CoM frame second-class constraints

$$\frac{dq_i^\mu}{d\tau} = \frac{p_i^\mu}{p_i^\mu U_\mu},$$

$$\frac{dp_i^\mu}{d\tau} = - \sum_{k=1}^2 \frac{1}{2p_k^\mu U_\mu} \frac{\partial \Phi_k}{\partial q_{i,\mu}},$$

RQMD/UrQMD

$$\frac{d\mathbf{q}_i}{d\tau} = \frac{\mathbf{p}_i^*}{p_i^{*0}} + \sum_{k=1}^2 \left[\frac{\mathbf{p}_k^*}{p_k^{*0}} \frac{\partial V_k}{\partial \mathbf{p}_i} + \frac{m_k^*}{p_k^{*0}} \frac{\partial m_k^*}{\partial \mathbf{p}_i} \right],$$

$$\frac{d\mathbf{p}_i}{d\tau} = - \sum_{k=1}^2 \left[\frac{\mathbf{p}_i^*}{p_i^{*0}} \frac{\partial V_{k,v}}{\partial \mathbf{q}_i} + \frac{m_k^*}{p_k^{*0}} \frac{\partial m_k^*}{\partial \mathbf{q}_i} \right].$$

JAM

Relativistic QMD

Trajectories with Vector or Scalar potential; w/o collisions

Density-dependent vector and scalar potentials:

$$A_i^\mu = \sum_{j \neq i}^N p_j^{*\mu} \rho_{ij}(q_T), \quad S_i = \sum_{j \neq i}^N m_j \rho_{ij}(q_T),$$

Test for 2-body case:

$$A_1^\mu = p_2^{*\mu} \rho_{12}(q_T),$$

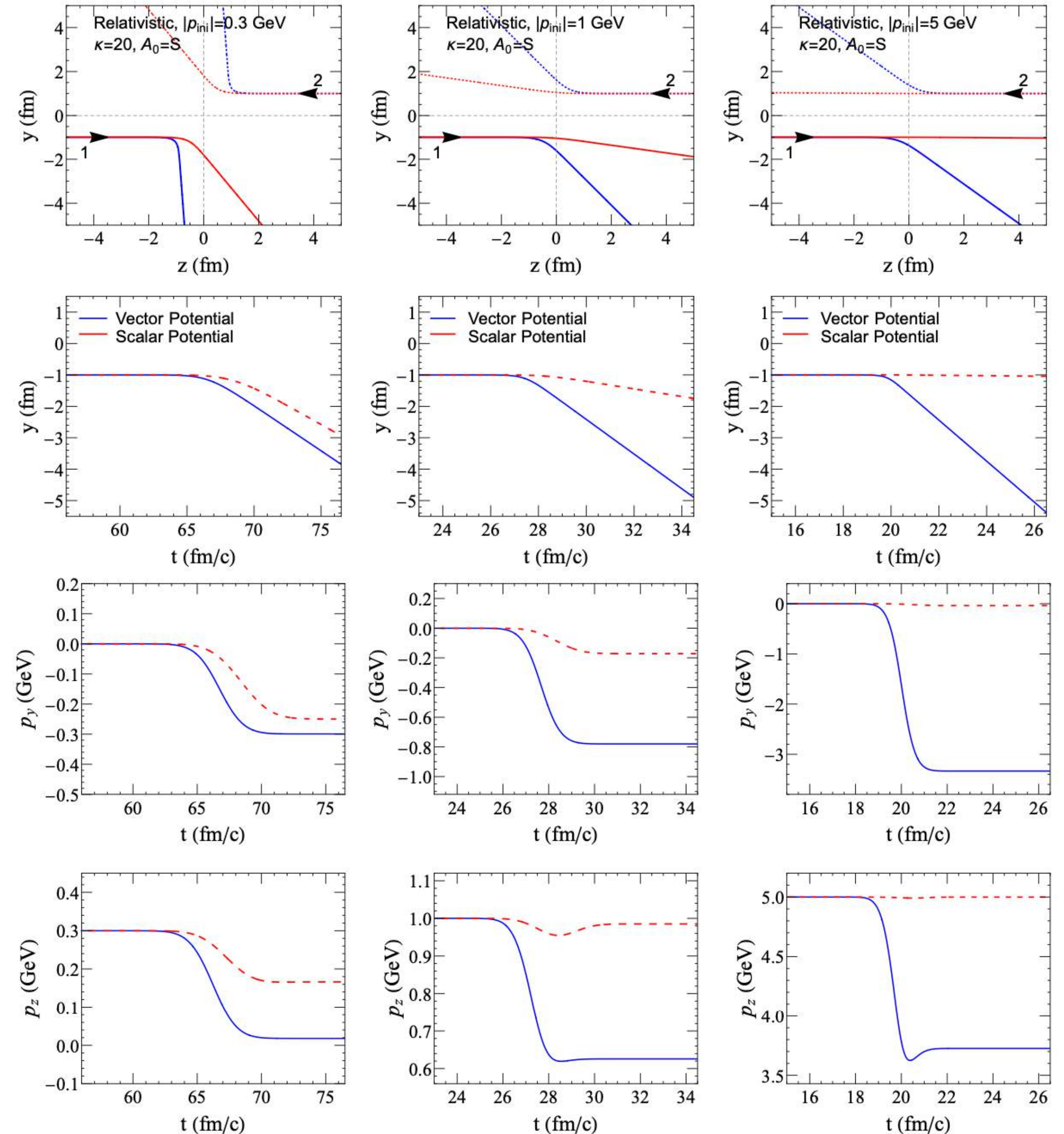
$$A_2^\mu = p_1^{*\mu} \rho_{12}(q_T),$$

$$\Phi_1 = \Phi_2 = 2\mu S = 2\mu m \rho_{12}(q_T),$$

$$\rho_{12}(q_T) = \frac{\kappa}{(2\pi L)^{3/2}} \exp(q_T^2 / (2L)),$$

Due to the “Lorentz force” in vector potential, **trajectories change more than** that with scalar potential \rightarrow **a larger v_1** .

PHRQMD is coming soon !



Summary

Based on the Kadanoff-Baym equation - the **PHSD** was developed to describe the **strongly-interaction hadronic and partonic matter** created in p+A and heavy-ion collisions from SIS to LHC energies.

PHQMD is a combination of the PHSD and QMD. It includes correlations and was developed to describe **nuclear cluster** and **hypernuclei productions** in heavy ion collisions from SIS to RHIC energies.

- ➔ **Cluster formation mechanism (excited states, internal structure,...)**
- ➔ **Relativistic QMD framework with both scalar and vector potentials for upcoming HIAF, NICA, FAIR energies**

Thank you !

Advantages of Kadanoff-Baym dynamics vs Boltzmann

Kadanoff-Baym equations:

- propagate two-point Green functions $G^<(x, p) \rightarrow A(x, p)N(x, p)$ in **8** dimensions
- $G^<$ carries information not only on the **occupation number** $N_{\chi P}$, but also on the particle properties, interactions and correlations via **spectral function** $A_{\chi P}$
→ **off-shell approach**

Boltzmann equations:

- propagate phase space distribution function $f(\vec{r}, \vec{p}, t)$ in **6+1** dimensions
- works well for small coupling = weakly interacting system,
→ **on-shell approach**

Kadanoff-Baym eqs. can be **solved exactly** for model cases such as **Φ^4 – theory**

Elementary hadronic interactions

Consider **all possible interactions – elastic and inelastic collisions -** for the system of (N,R,m) , where N -nucleons, R - resonances, m -mesons, and **resonance decays**

Low energy collisions:

- binary $2 \leftrightarrow 2$ and $2 \leftrightarrow 3(4)$ reactions
- $1 \leftrightarrow 2$: formation and **decay** of baryonic and mesonic resonances

$BB \leftrightarrow B'B'$

$BB \leftrightarrow B'B'm$

$mB \leftrightarrow m'B'$

$mB \leftrightarrow B'$

$mm \leftrightarrow m'm'$

$mm \leftrightarrow m' \dots$

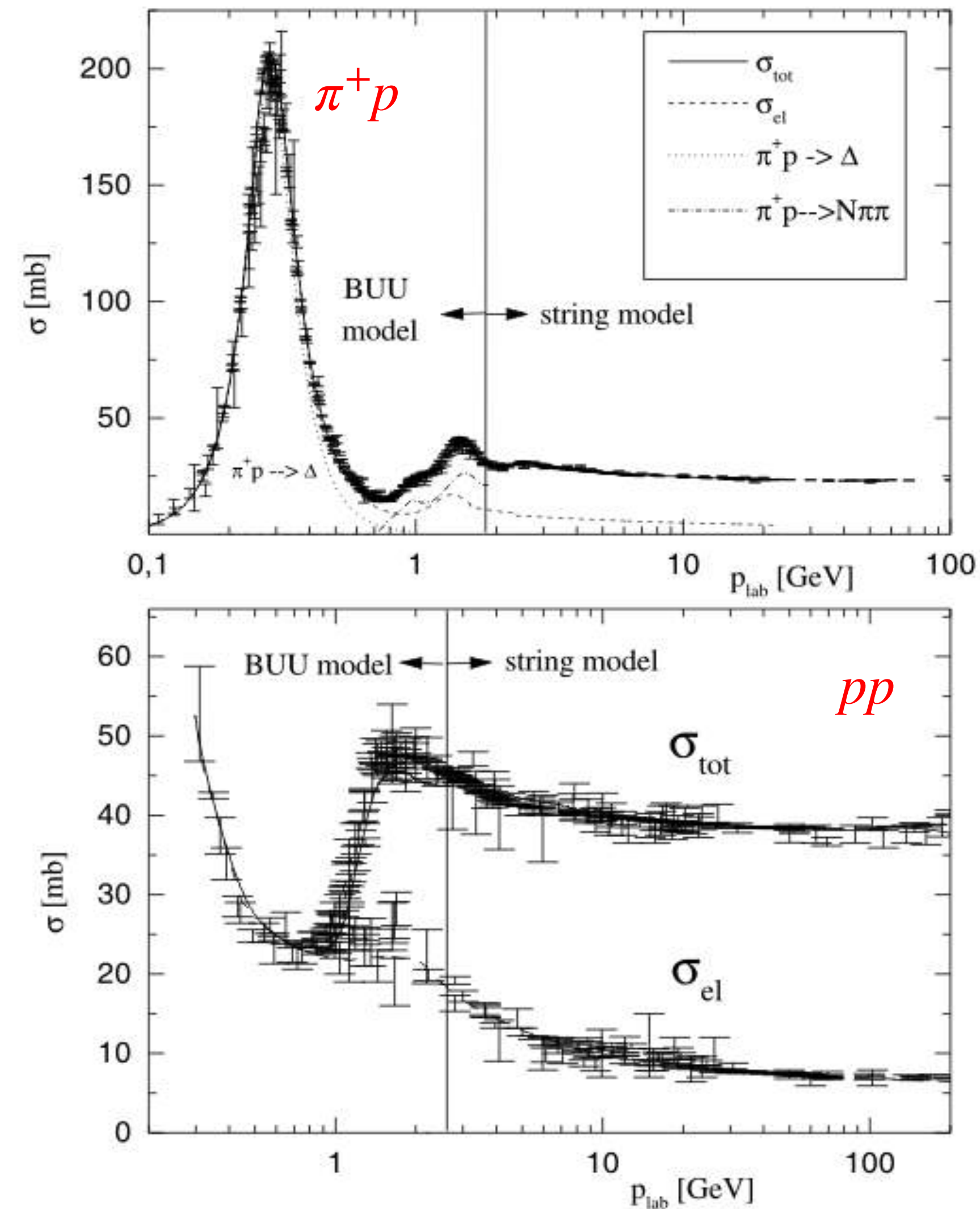
Baryons:

$B = p, n, \Delta(1232), N(1440), N(1535),$

...

Mesons:

$M = \pi, \eta, \rho, \omega, \phi, \dots$



Collision term in off-shell transport approach

Collision term for reaction $1+2 \rightarrow 3+4$:

(Generalized off-shell collision integral for $n \leftrightarrow m$ reactions:)

W. Cassing, NPA 700 (2002) 618

$$I_{coll}(X, \vec{P}, M^2) = \text{Tr}_2 \text{Tr}_3 \text{Tr}_4 \underbrace{A(X, \vec{P}, M^2) A(X, \vec{P}_2, M_2^2) A(X, \vec{P}_3, M_3^2) A(X, \vec{P}_4, M_4^2)}_{\text{spectral functions}}$$

$$\underbrace{|G((\vec{P}, M^2) + (\vec{P}_2, M_2^2) \rightarrow (\vec{P}_3, M_3^2) + (\vec{P}_4, M_4^2))|_{A,S}^2}_{\text{Gain term}} \delta^{(4)}(P + P_2 - P_3 - P_4)$$

$$\left[\underbrace{N_{X\vec{P}_3M_3^2} N_{X\vec{P}_4M_4^2} \bar{f}_{X\vec{P}M^2} \bar{f}_{X\vec{P}_2M_2^2}}_{\text{Gain term}} - \underbrace{N_{X\vec{P}M^2} N_{X\vec{P}_2M_2^2} \bar{f}_{X\vec{P}_3M_3^2} \bar{f}_{X\vec{P}_4M_4^2}}_{\text{Loss term}} \right]$$

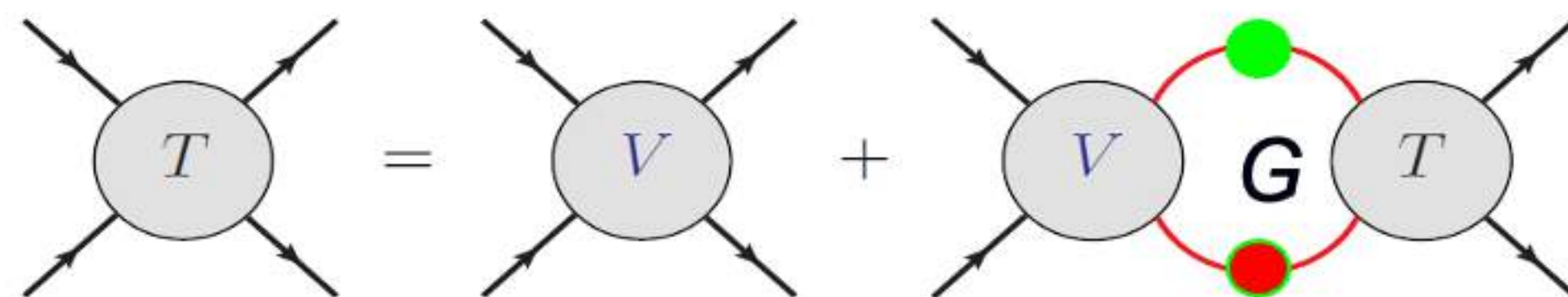
$$\bar{f}_{X\vec{P}M^2} = 1 + \eta N_{X\vec{P}M^2} \quad \eta = 1 \text{ for bosons, } -1 \text{ for fermions}$$

The trace over particles 2,3,4 reads explicitly

fermions: $Tr_2 = \sum_{\sigma_2, \tau_2} \frac{1}{(2\pi)^4} \int d^3P_2 \frac{dM_2^2}{2\sqrt{\vec{P}_2^2 + M_2^2}}$

bosons: $Tr_2 = \sum_{\sigma_2, \tau_2} \frac{1}{(2\pi)^4} \int d^3P_2 \frac{dP_{0,2}^2}{2}$

The off-shell transport approach and the particle spectral functions are fully determined once the **in-medium transition amplitudes G** are known in their **off-shell dependence!**



Numerical solution of BUU EoM

Testparticle method or **method of parallel ensembles** :
 the 1-body phase space distribution function is described as a sum of N point-like particles (δ –functions).

In the limit of large number of parallel ensembles $N_t \rightarrow \infty$

$$f(\vec{r}, \vec{p}, t) = \frac{1}{N_t} \sum_{i=1}^{N \cdot N_t} \delta(\vec{r} - \vec{r}_i(t)) \delta(\vec{p} - \vec{p}_i(t)) \quad \text{is a solution of Vlasov EoM}$$

$$\frac{1}{N_t} \left(\begin{array}{c} \text{1} \\ \text{2} \\ \text{3} \\ \dots \\ \text{N} \end{array} \right)$$

- **Testparticle method** provides a smooth density distribution for calculation of **mean-field potential** for particle propagation.
 - No exchange of particles between the parallel ensembles, particles collide only inside one ensemble

→ **Propagation of test-particles** in time following '**classical**' EoM:

$$\dot{\vec{r}}_i = \frac{d\vec{r}_i}{dt} = \frac{\vec{p}_i}{m_i}$$

$$\dot{\vec{p}}_i = \frac{d\vec{p}_i}{dt} = -\vec{\nabla}_{\vec{r}_i} U(\vec{r}_i, t)$$

