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# PHSD/PHQMD for FAIR

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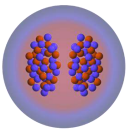
for the PHSD/PHQMD team



45th CBM Collaboration Meeting,  
GSI, 19 February 2025



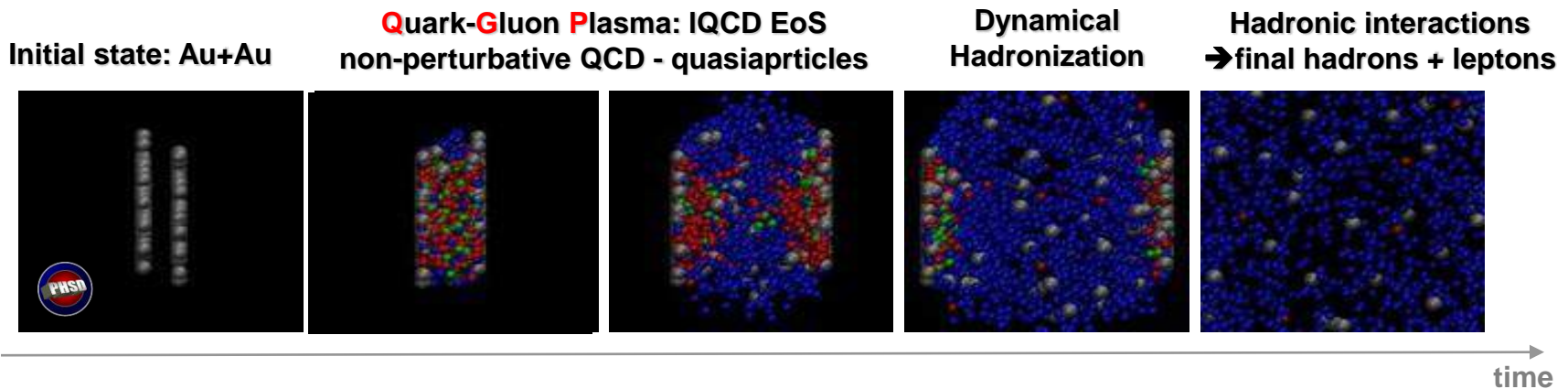
# Study of QCD matter with heavy-ions



Goal: Microscopic modeling of heavy-ion collisions

**PHSD & PHQMD**  
Parton-Hadron-String Dynamics & Parton-Hadron-Quantum-Molecular Dynamics

is a **unified non-equilibrium microscopic transport approach** for the description of the dynamics of strongly-interacting **hadronic and partonic matter** created in heavy-ion collisions and  $p+A$ ,  $p+p$ ,  $\pi+A$  reactions from SIS to LHC energies



→ provides a **continuous description of the HIC dynamics**:  
– no artificial transition from micro- to macro-description  
as in hydro-type models, no jump in entropy and energy density



PHSD, PHQMD are the open source codes, available for all experimental collaborations (used by GSI/FAIR collaborations, linked to the exp. software) and upon registration for other users

# PHSD/PHQMD code



**PHSD mode**

**PHQMD mode**

Initialization A+A  
+ propagation of **baryons**:  
Mean Field dynamics  
(BUU)

Initialization A+A  
+ propagation of **baryons**:  
Quantum Molecular dynamics  
(QMD) – n-body model

Propagation of partons (quarks, gluons) and mesons:  
Mean Field dynamics (Kadanoff-Baym, BUU)

 **Collision integral** = interactions of hadrons and partons (QGP)

*Optionally*  
Cluster recognition: **MST** (Minimum Spanning Tree)  
or **SACA** (Simulated Annealing Clusterization Algorithm)  
or coalescence mechanism + kinetic deuterons

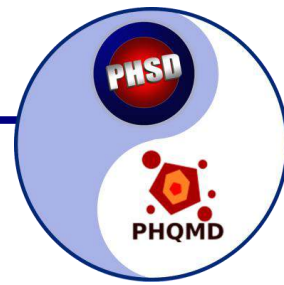


Final output – “events” : OSCAR, ROOT, Rivet formats

Realization: parallel ensemble method

Computer language: Fortran

# Special features of PHSD/PHQMD



- ✓ Kadanoff-Baym dynamics
  - ✓ **in-medium effects** within G-matrix
  - ✓ **EoS in MF and QMD:** momentum dependent potential
  - ✓ **Mechanisms for cluster production in PHQMD:**  
dynamical, kinetic and coalescence
  - ✓ **QGP** within **DQPM** → **PHSD**;  
phase transition from hadronic matter to QGP
  - ✓ Dark matter – dark photons
  - Realization of  $n \leftrightarrow m$  reactions
  - **chiral symmetry restoration** via Schwinger mechanism
  - Charm/bottom dynamics
- ✓ **More information on these topics are in Backup slides**

Dynamics of heavy-ion collisions is a **many-body problem!**

**Schrödinger eq. for system of N particles**  $i\hbar \frac{\partial}{\partial t} \Psi(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N, t) = H(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N, t) \Psi(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N, t)$

**N-body Hamiltonian:**

$$H \approx \sum_{i=1}^N T(\vec{r}_i) + \sum_{i < j}^N V_{ij}(\vec{r}_1 - \vec{r}_2, t)$$

**local two-body potential**

**N-body wave function:**

$$\Psi(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N, t) \approx \prod_{i=1}^N \psi_i(\vec{r}_i, t)$$

## Mean field dynamics

**Hartree-Fock eq. for a single particle i:**

$$i\hbar \frac{\partial}{\partial t} \psi_i(\vec{r}, t) = \hat{h} \psi_i(\vec{r}, t)$$

$h = T+U$  - single particle Hamiltonian  
 $\psi_i$  - single particle wave function

**Self-generated mean-field potential:**

$$U(\vec{r}, t) = \frac{1}{(2\pi\hbar)^3} \int d^3r' d^3p V(\vec{r} - \vec{r}', t) f(\vec{r}', \vec{p}, t)$$

**local two-body potential**

**Testparticle or parallel ensembles method:**

$$f(\vec{r}, \vec{p}, t) = \frac{1}{N_t} \sum_{i=1}^{N \cdot N_t} \delta(\vec{r} - \vec{r}_i(t)) \delta(\vec{p} - \vec{p}_i(t))$$

**MF- covariant**  $\frac{1}{N_t} \left( \begin{matrix} \text{1} \\ \text{2} \\ \text{3} \\ \dots \\ \text{N} \end{matrix} \right)$

## QMD dynamics

**Ritz variational principle:**

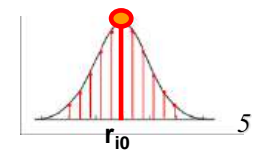
$$\delta \int_{t_1}^{t_2} dt \langle \psi(t) | i \frac{d}{dt} - H | \psi(t) \rangle = 0$$

$H$  - N-body Hamiltonian  
 $\Psi$  - N-body wave function

**Single-particle Wigner density of the nucleon wave function  $\psi_i$ :**

$$f(\mathbf{r}_i, \mathbf{p}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t) = \frac{1}{\pi^3 \hbar^3} e^{-\frac{2}{L} [\mathbf{r}_i - \mathbf{r}_{i0}(t)]^2} e^{-\frac{L}{2\hbar^2} [\mathbf{p}_i - \mathbf{p}_{i0}(t)]^2}$$

**Ansatz: Gaussian trial wave function with width L centered at  $r_{i0}, p_{i0}$**   
**→ QMD - non-covariant !**



## Mean field dynamics

1 event: A+A nucleons

→  $N_t$  ensembles with  $(A+A) \cdot N_t$  test particles

### Step 1: grid cells

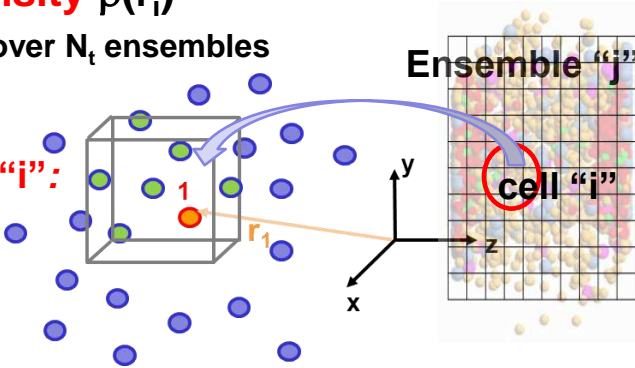
$$\frac{1}{N_t} \left( \text{ensemble 1} + \text{ensemble 2} + \text{ensemble 3} + \dots + \text{ensemble } N_t \right)$$

- **local density**  $\rho(\mathbf{r}_i)$

- averaged over  $N_t$  ensembles

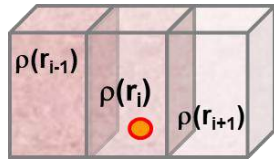
**Local cell "i":**

$$\Delta x = \Delta y = 1 \text{ fm}, \\ \Delta z = 1 \text{ fm}/\gamma$$



### Step 2: forces

$$\vec{F} = -\vec{\nabla}U \sim -\vec{\nabla}\rho \frac{\partial U}{\partial \rho}$$



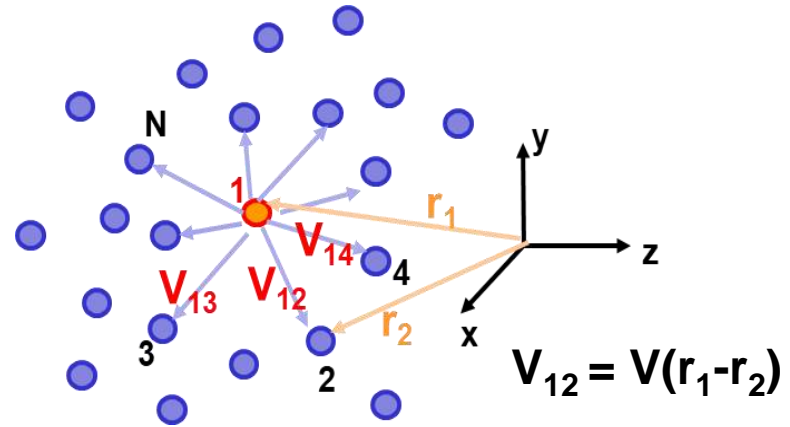
$$\dot{\vec{r}}_i = \frac{d\vec{r}_i}{dt} = \frac{\vec{p}_i}{m_i}$$

$$\dot{\vec{p}}_i = \frac{d\vec{p}_i}{dt} = -\vec{\nabla}_{\vec{r}_i} U(\vec{r}_i, t)$$

$U(\vec{r}, t)$  - **mean-field potential**

## QMD dynamics

1 event: A+A nucleons



Expectation value of N-body Hamiltonian:

$$\langle H \rangle = \sum_i \langle H_i \rangle = \sum_i \left( \langle T_i \rangle + \sum_{j \neq i} \langle V_{i,j} \rangle \right)$$

$$\dot{r}_{i0} = \frac{\partial \langle H \rangle}{\partial p_{i0}} \quad \dot{p}_{i0} = -\frac{\partial \langle H \rangle}{\partial r_{i0}}$$

$V_{ij} = V(\mathbf{r}_i - \mathbf{r}_j)$  - **local two-body potential**

→ **QMD: N-body dynamics**

via 2-body interaction potential

# Where do we see differences in QMD vs MF dynamics?

- “Bulk” observables for hadrons are rather similar in QMD and MF!  
→ tested with PHSD/PHQMD framework
- **Cluster formation** is sensitive to nucleon dynamics:
  - **QMD** – allows to keep over time NN correlations by potential interaction
  - **MF** – correlations are smeared out
  - **Cascade** – no correlations by potential interactions

Example: **Cluster stability over time:**

Viktar Kireyeu, Phys.Rev.C 103 (2021) 5

- Clusters identified by psMST for all models:

**QMD:**

— PHQMD + psMST

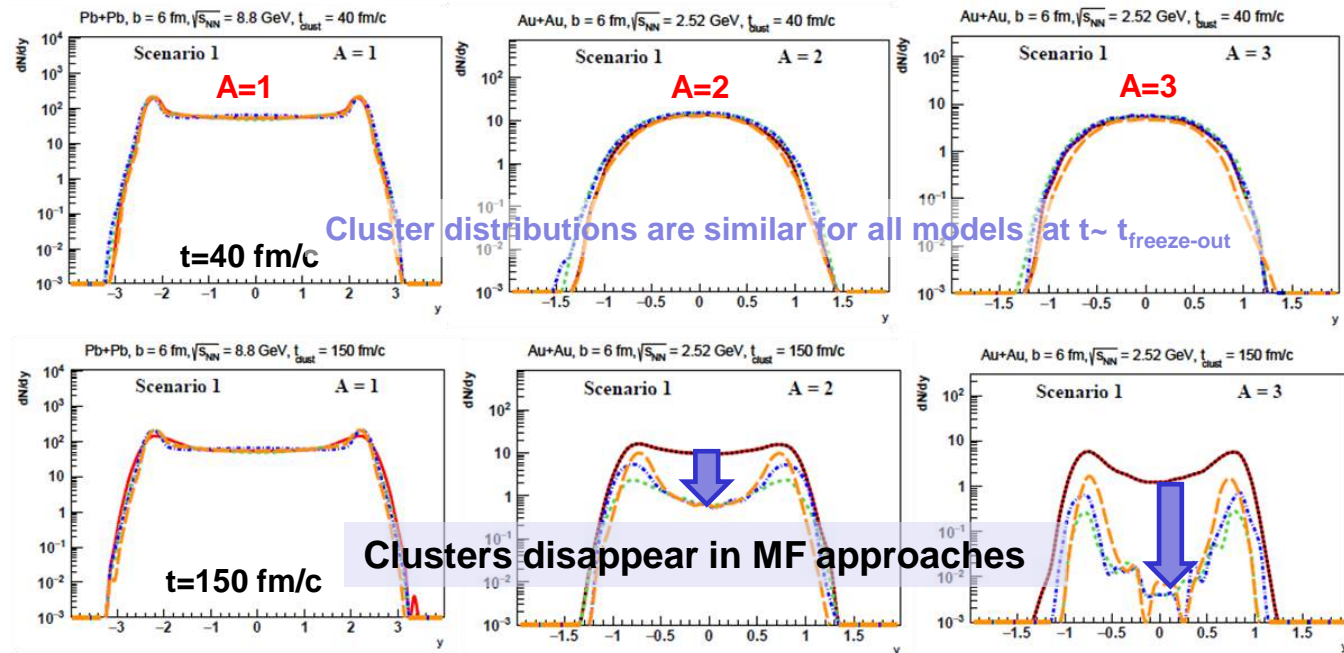
**MF:**

— PHSD + psMST

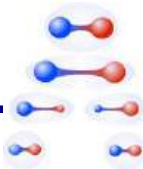
**Cascade:**

■ SMASH + psMST

■ UrQMD + psMST



# Elementary hadronic interactions in PHSD/PHQMD

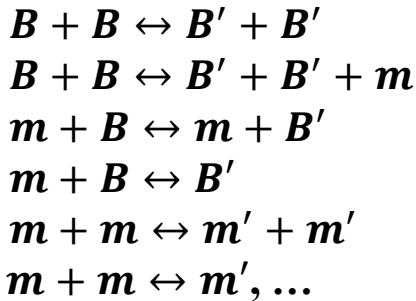


## Hadronic degrees-of-freedom:

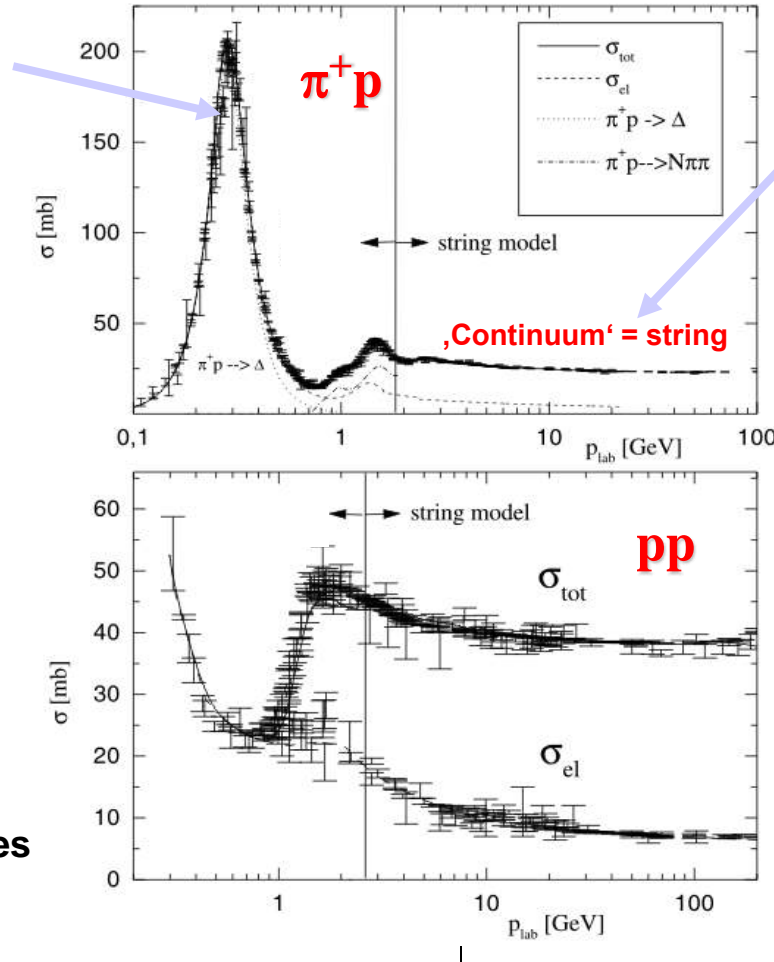
- baryons (21 states):  $B = p, n, \Delta(1232), N(1440), N(1535), \dots$
- mesons (37 states):  $M = \pi, \eta, \rho, \omega, \phi, \dots$

## Low energy reactions

- elastic and inelastic:  $2 \leftrightarrow 2, 2 \leftrightarrow 3(4)$
- resonance formation and decay  $1 \leftrightarrow 2$



\* Modeling of in-medium hadronic interactions for strongly interacting particles (off-shell dynamics)



## High energy collisions:

(above  $s^{1/2} \sim 2.4$  GeV)

Inclusive particle production:

$$BB \rightarrow X, mB \rightarrow X, mm \rightarrow X$$

$X$  = many particles

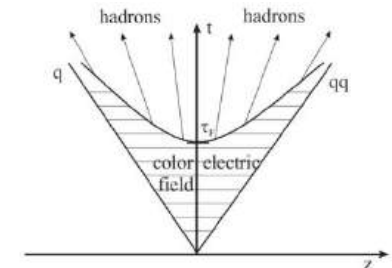
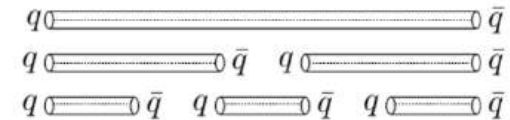
described by

string formation and decay

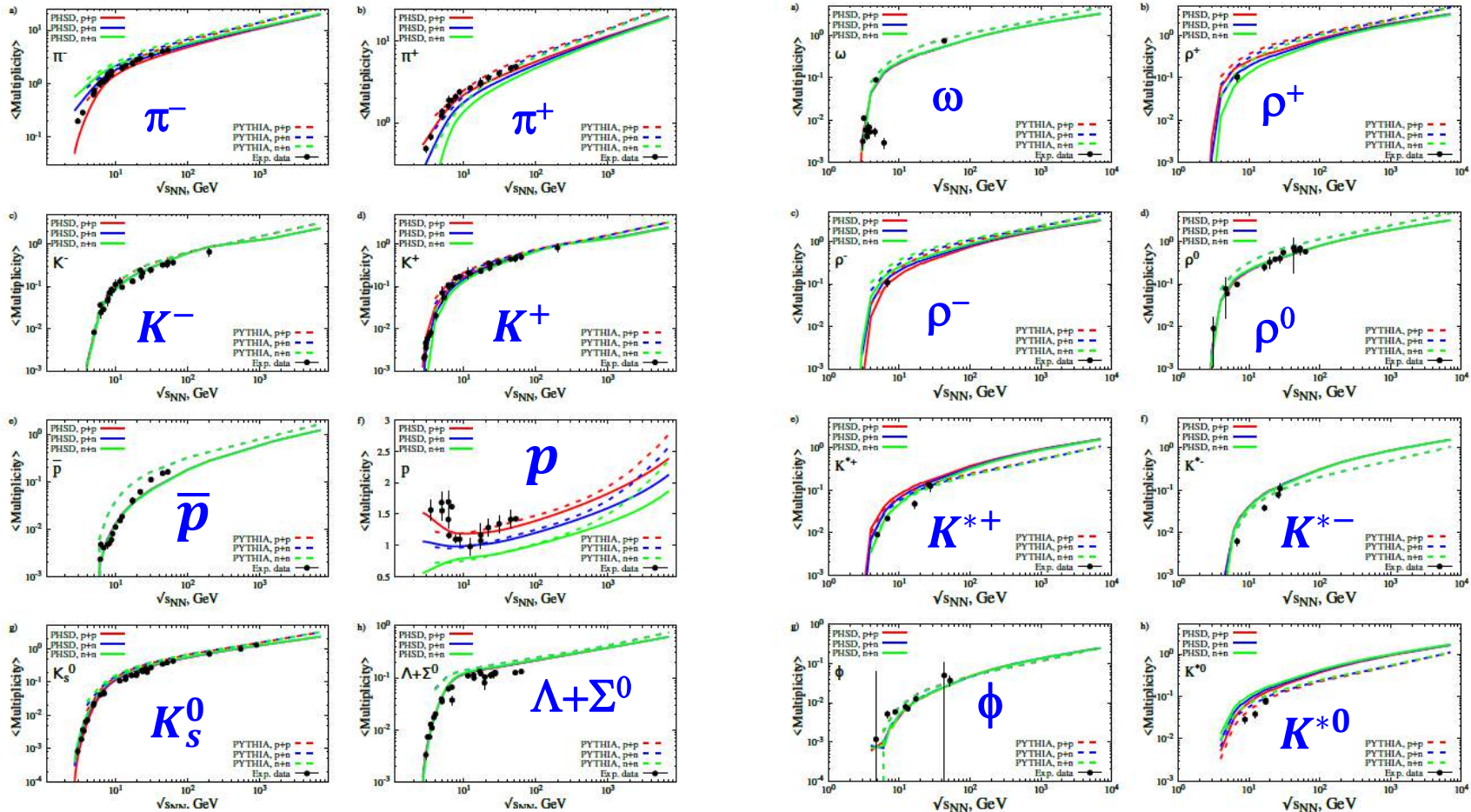
using LUND string model:

strings = excited color singlet states  $q\text{-}q\bar{q}, q\text{-}q\bar{q}$

→ color flux tube



- LUND string model (FRITIOF, PYTHIA 6.4) with **PHSD tune** vs **PYTHIA 8.2**



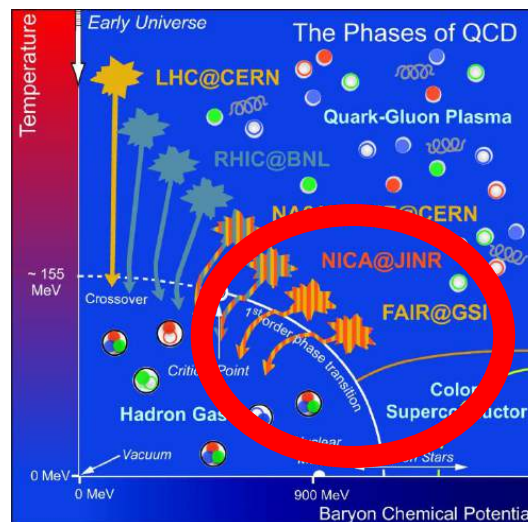
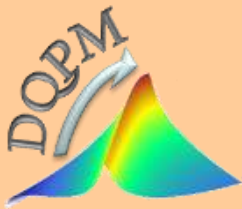
- Existing experimental data on p+p are poor
- Practically **NO** data on p+n reactions

➔ **FAIR data on elementary reactions are needed!**

- Poor data on resonance production

# Modeling of sQGP in microscopic transport theory:

## DQPM ( $T, \mu_q$ ) in PHSD/PHQMD

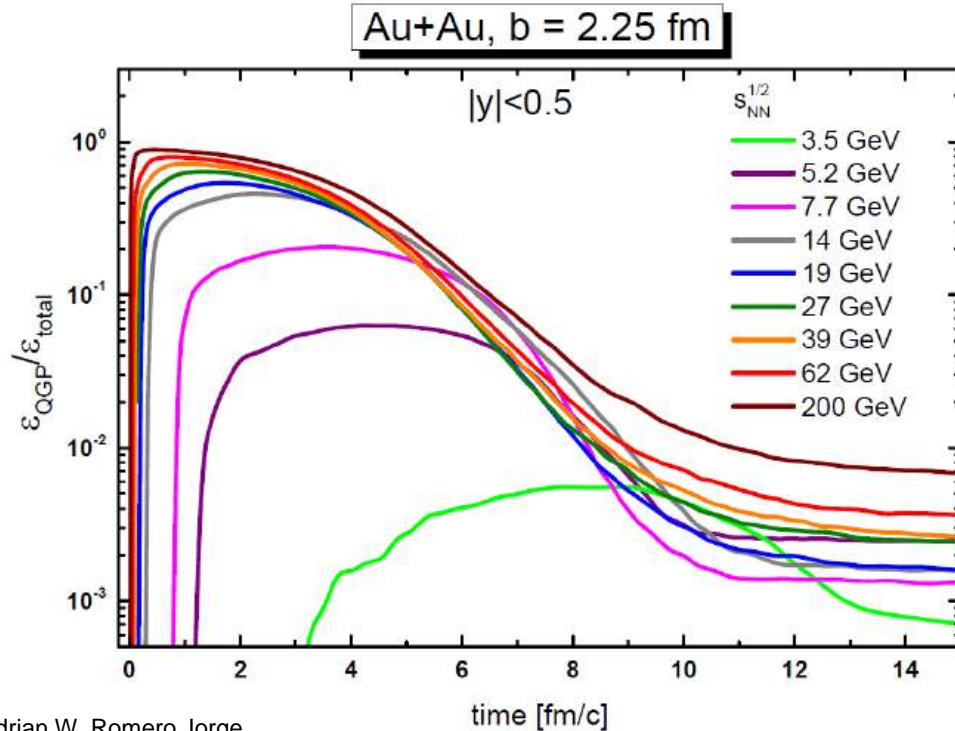


**finite  $\mu_q$**



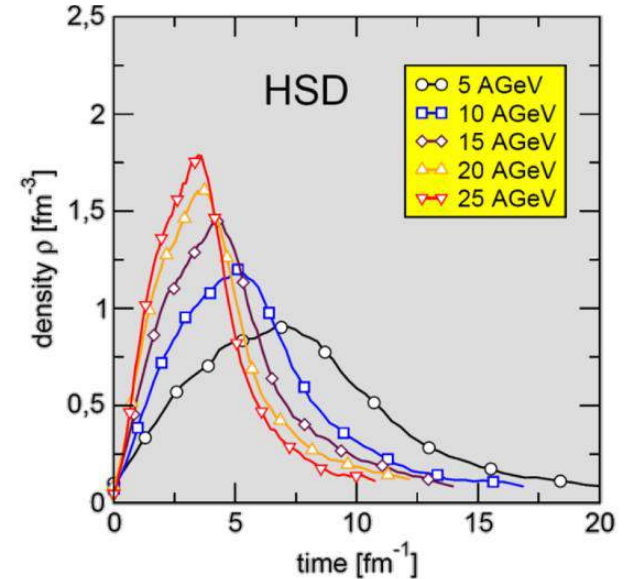
# Dense and hot matter created in HICs

## Time evolution of the partonic energy fraction



Plot by Adrian W. Romero Jorge

## Time evolution of the baryon density $\rho$

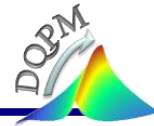


Large energy and baryon densities (above critical  $\epsilon > \epsilon_{\text{crit}} \sim 0.4$  GeV/fm<sup>3</sup>) are reachable in central reaction region at **CBM energies\***

→ phase transition from **hadronic matter** to **QGP**

\* small volume of QGP (**droplets**) at low energies

# QGP: Dynamical QuasiParticle Model (DQPM)



**DQPM** – effective model for the description of **non-perturbative** (strongly interacting) QCD based on **IQCD EoS**

**Degrees-of-freedom**: strongly interacting **dynamical quasiparticles** - quarks and gluons

## Theoretical basis :

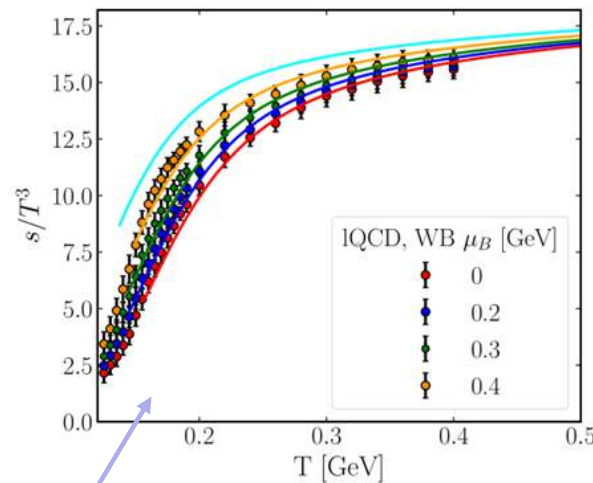
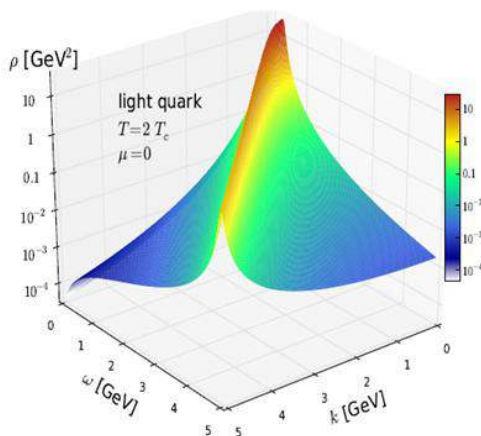
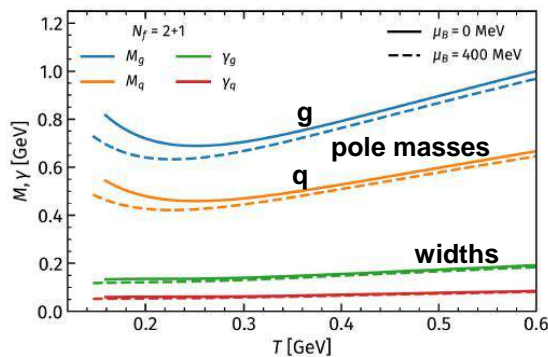
□ ,resummed' single-particle Green's functions  $\rightarrow$  quark (gluon) propagator (2PI) :  $G_q^{-1} = P^2 - \Sigma_q$

**Properties of the quasiparticles** are specified by scalar **complex self-energies**:  $\Sigma_q = M_q^2 - i2\gamma_q\omega$

$Re\Sigma_q$  : **thermal masses** ( $M_g, M_q$ );  $Im\Sigma_q$  : **interaction widths** ( $\gamma_g, \gamma_q$ )  $\rightarrow$  spectral functions  $\rho_q = -2ImG_q$

- introduce an **ansatz** (HTL; with few parameters) for the ( $T, \mu_B$ ) dependence of masses/widths
- evaluate the **QGP thermodynamics** in equilibrium using the Kadanoff-Baym theory
- fix DQPM parameters by comparison of the DQPM entropy density to **IQCD** at  $\mu_B=0$

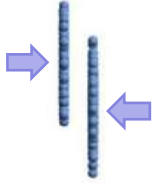
## $\rightarrow$ Quasi-particle properties at ( $T, \mu_B$ ) :



$\rightarrow$  very good agreement with IQCD data for QGP thermodynamics at finite ( $T, \mu_B$ )

• **DQPM** provides **mean-fields** (1PI) for q,g and **effective 2-body partonic interactions** (2PI); gives **transition rates** for the hadronization  $\rightarrow$  **sQGP in PHSD**

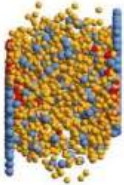
Initial A+A collision



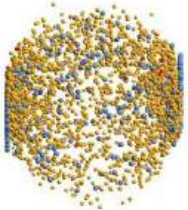
Partonic phase



Hadronization



Hadronic phase



Initial A+A collisions :  
 $N+N \rightarrow$  string formation  $\rightarrow$  decay to pre-hadrons + leading hadrons

Formation of QGP stage if local  $\epsilon > \epsilon_{\text{critical}}$  :  
 dissolution of pre-hadrons  $\rightarrow$  partons

Partonic phase - QGP:  
 QGP is described by the **Dynamical QuasiParticle Model (DQPM)** matched to reproduce **lattice QCD EoS** for finite T and  $\mu_B$  (crossover)

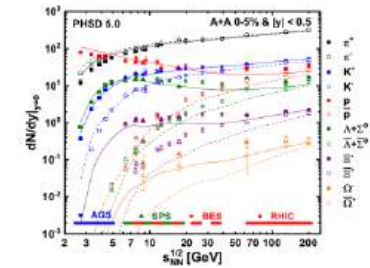
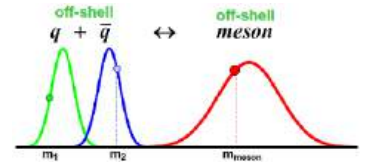
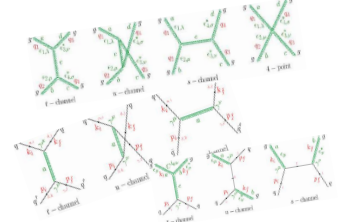
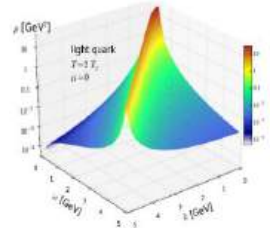
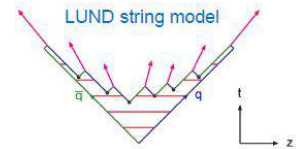
- **Degrees-of-freedom**: strongly interacting quasiparticles:  
**massive quarks and gluons ( $g, q, q_{\text{bar}}$ )** with sizeable collisional widths in a self-generated mean-field potential

- **Interactions**: (quasi-)elastic and inelastic collisions of partons  
 - parton propagation within **Kadanoff-Baym equations**

Hadronization to colorless **off-shell mesons and baryons**:  
 Strict 4-momentum and quantum number conservation

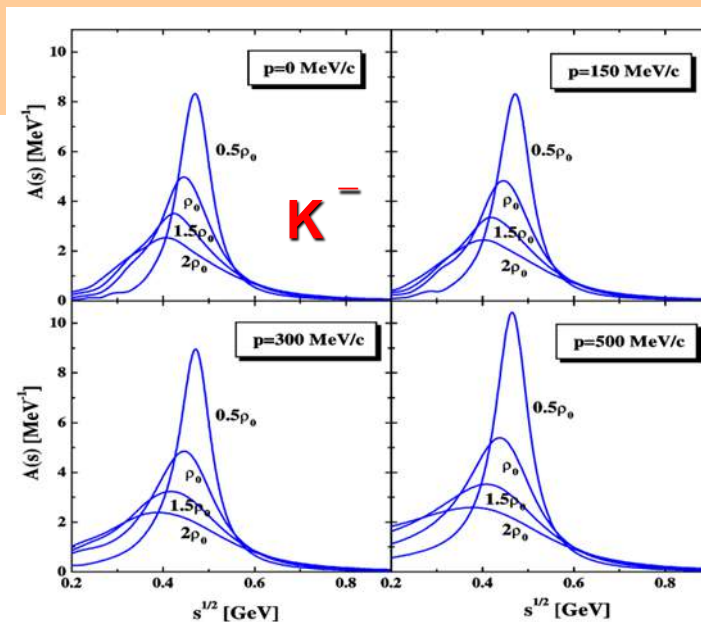
Hadronic phase: **hadron-hadron interactions – off-shell HSD** including  $n \leftrightarrow m$  selected reactions (for strangeness, anti-baryons, deuteron production)

$\rightarrow$  PHSD/PHQMD provides a **good description of ‘bulk’ hadronic and electromagnetic observables** from SIS to LHC energies



# Strongly interacting hadrons (vector mesons, strange mesons) in hot and dense medium:

## from BUU to Kadanoff-Baym



# From weakly to strongly interacting systems

**In-medium effects** (on hadronic or partonic levels) = changes of particle properties in hot and dense medium

Examples: **hadronic medium** - vector mesons, strange mesons, baryons  
**QGP** – dressing of partons

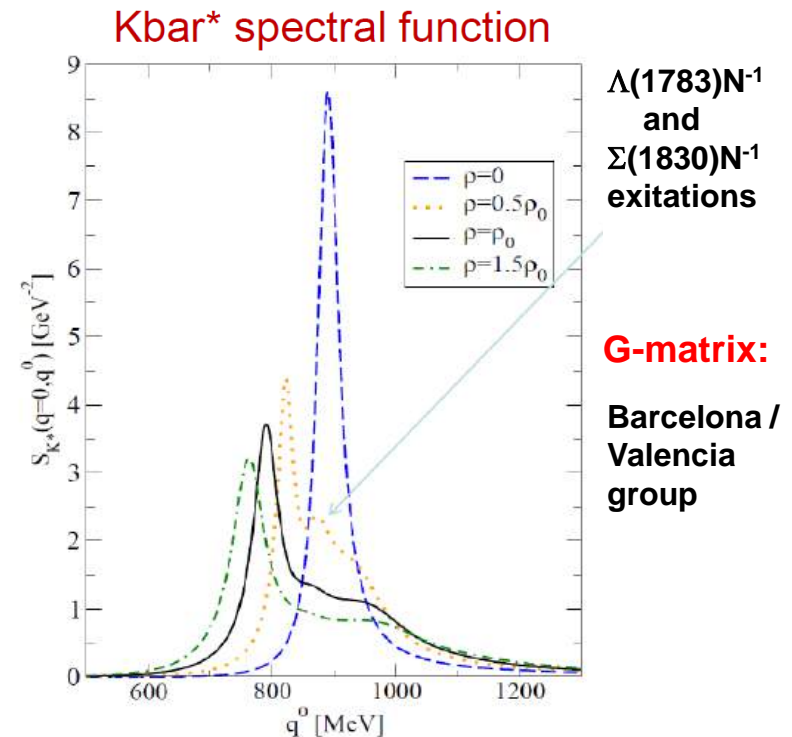
Many-body theory:

**Strong interaction** → large widths → broad spectral functions → **quantum objects**

**Semi-classical on-shell BUU:** applies for a weakly interacting systems of particles

▪ How to describe the dynamics of broad **strongly interacting quantum states** in transport theory?

It is doable with **quantum Kadanoff-Baym equations**



# Off-shell propagation: Kadanoff-Baym

After the first order gradient expansion of the Wigner transformed Kadanoff-Baym equations and separation into the real and imaginary parts one gets:

## Generalized transport equations (GTE):

$$\underbrace{\diamond \{ P^2 - M_0^2 - \text{Re} \Sigma_{XP}^{\text{ret}} \}}_{\text{drift term}} \underbrace{\{ S_{XP}^< \}}_{\text{Vlasov term}} - \underbrace{\diamond \{ \Sigma_{XP}^< \} \{ \text{Re} S_{XP}^{\text{ret}} \}}_{\text{backflow term}} = \frac{i}{2} [ \Sigma_{XP}^> S_{XP}^< - \Sigma_{XP}^< S_{XP}^> ]$$

collision term = ,gain' - ,loss' term

off-shell behavior

W. Botermans, R. Malfliet,  
Phys. Rep. 198 (1990) 115

- KB propagates 1-body 2-points Green functions  $S^<(x,p) \rightarrow A(x,p) * N(x,p)$  in 8 dimensions
- $S^<$  carries information not only on the **occupation number**  $N_{XP}$  (as BUU), but also on the particle properties, interactions and correlations via **spectral function**  $A_{XP}$

**Spectral function:**

$$A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - \text{Re} \Sigma_{XP}^{\text{ret}})^2 + \Gamma_{XP}^2/4}$$

**Reaction rate of particle:**

$$\Gamma_{XP} = -\text{Im} \Sigma_{XP}^{\text{ret}} = 2 p_0 \Gamma$$

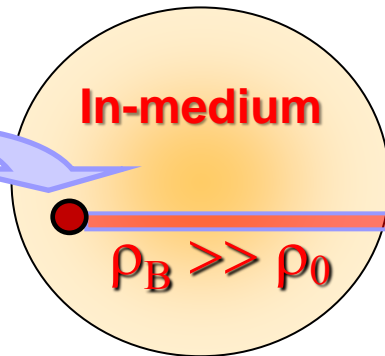
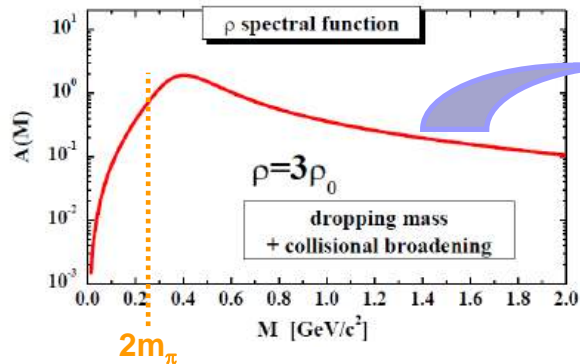
**On-shell limit of KB:**

- $A_{XP} \rightarrow \delta(p^2 - M^2)$  or  $A_{XP}$  has a constant shape in a medium, i.e.  $\nabla_X \Gamma = 0$ ,  $\nabla_P \Gamma = 0$
- **backflow term vanishes: KB  $\rightarrow$  BUU**

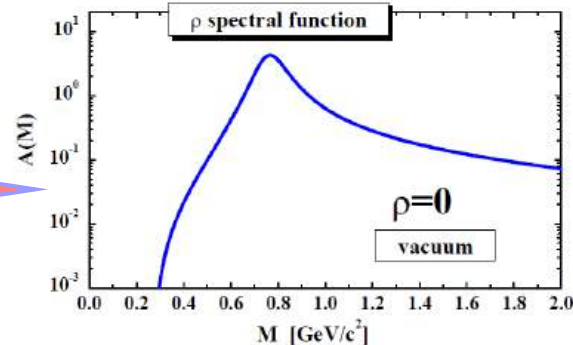
**$\rightarrow$  Generalized Cassing-Juchem off-shell equations of motion for testparticles  $\rightarrow$  PHSD**

# Off-shell vs. on-shell transport dynamics

**In-medium:  
production of broad state**

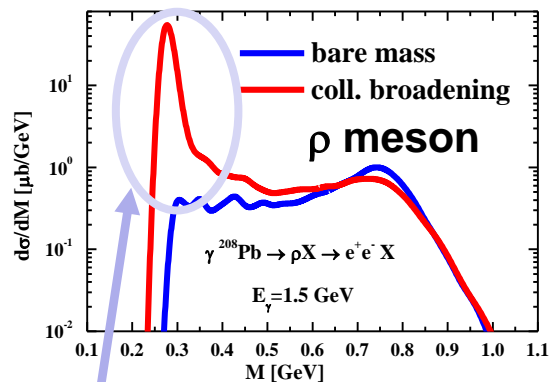


**Vacuum ( $\rho = 0$ ) state**



**Mass distribution of  $\rho$  mesons vs time**

1999: dilepton spectra - if one propagates in-medium  $\rho$  with on-shell BUU:

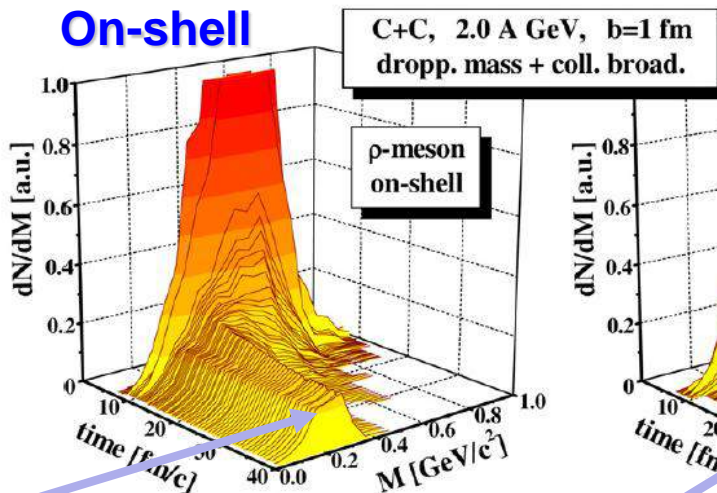


**On-shell BUU:**

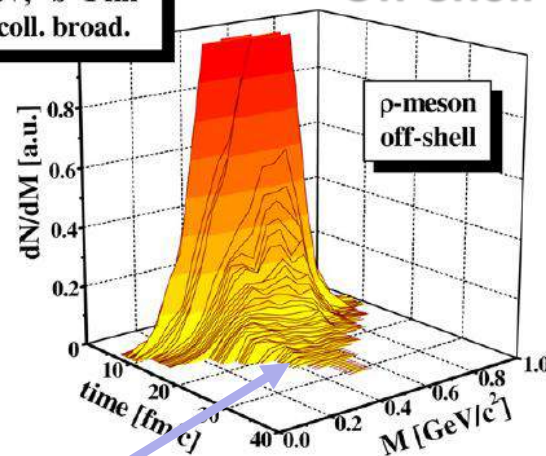
low mass  $\rho$  and  $\omega$  mesons live forever (and shine ,fake' dileptons)!

(e)GiBUU: M. Effenberger et al, PRC60 (1999) 027601

**On-shell**



**Off-shell**



EB & Cassing, NPA 807 (2008) 214

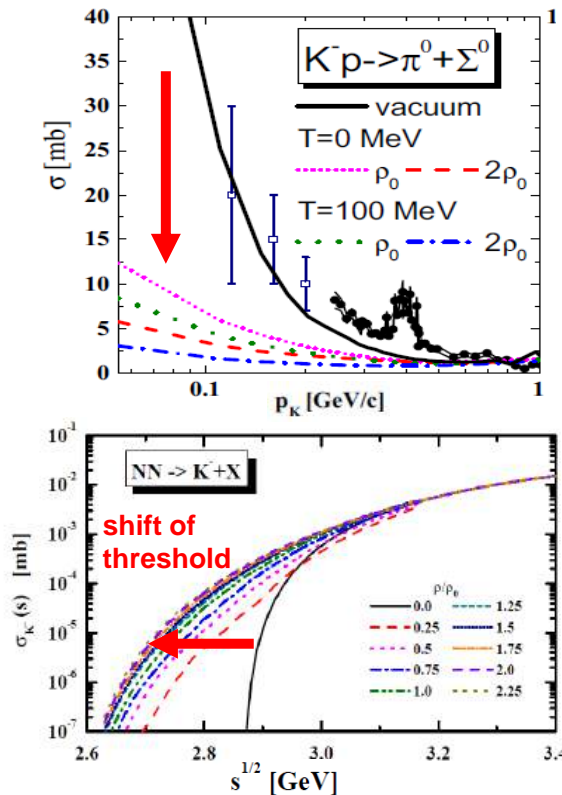
The **off-shell** spectral function comes to the **vacuum** shape **dynamically** by propagation through the medium!

# Off-shell dynamics for antikaons at SIS energies

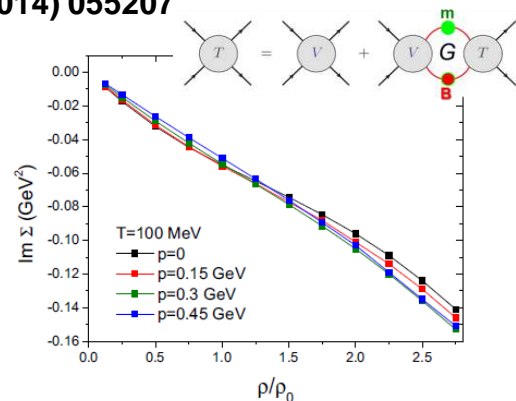
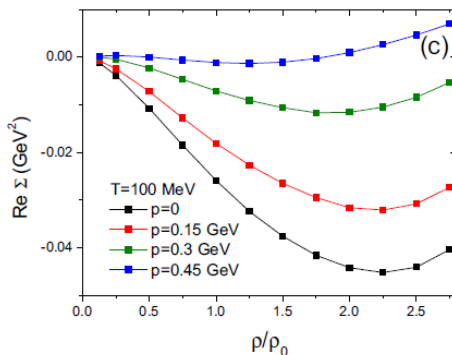
Spectral function of  $K^-$  within the **G-matrix** approach:

$$S_{\bar{K}}(k_0, \vec{k}; T) = -\frac{1}{\pi} \frac{\text{Im} \Sigma_{\bar{K}}(k_0, \vec{k}; T)}{|k_0^2 - \vec{k}^2 - m_{\bar{K}}^2 - \Sigma_{\bar{K}}(k_0, \vec{k}; T)|^2}$$

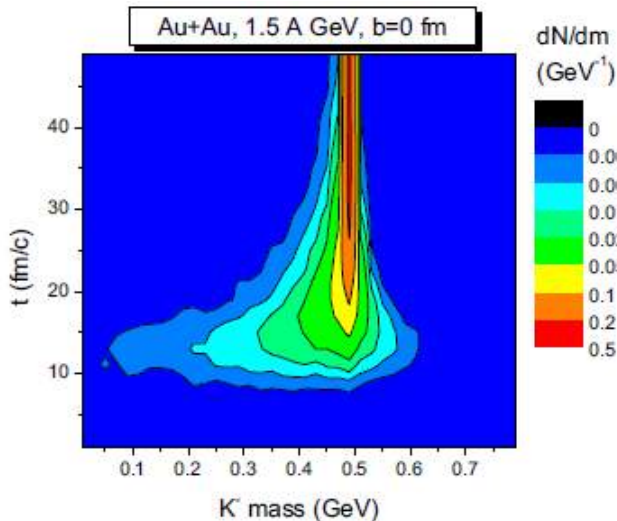
In-medium cross sections for  $K^-$  production and absorption are strongly modified in the medium:



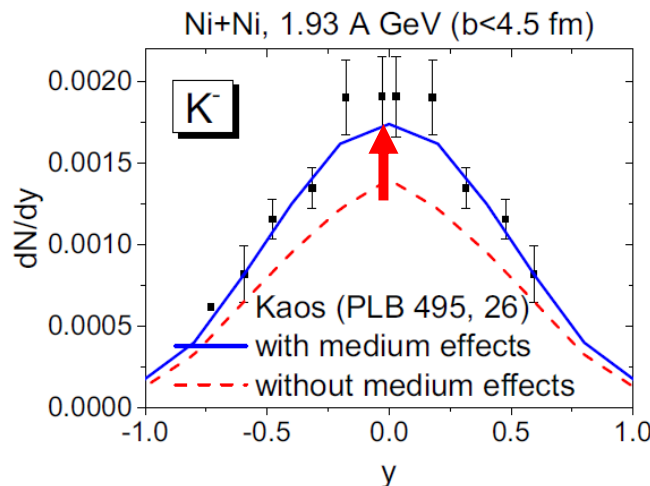
D. Cabrera et al., PRC 90 (2014) 055207



Time evolution of the  $K^-$  masses



In-medium effects are mandatory for the description of experimental  $K^-$  spectra

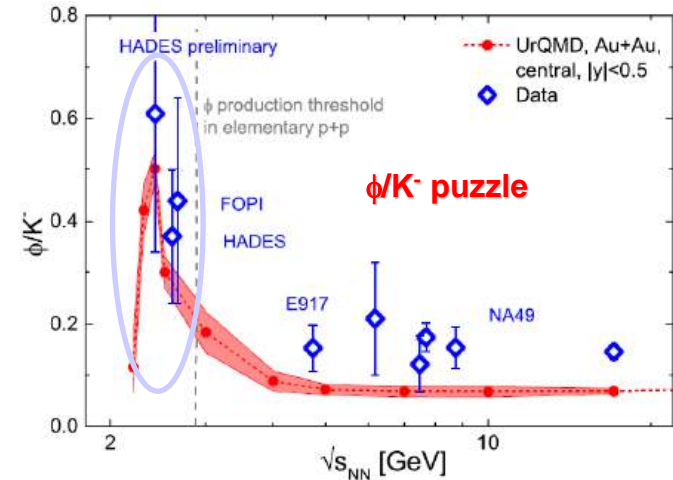


# In-medium properties of $\phi$ mesons

Experiment:

**strong enhancement of  $\phi/K^-$  ratio at low  $\sqrt{s}$**

UrQMD, SMASH:  $\phi/K^-$  ratio is qualitatively reproduced due to the **multi-step processes** for heavy baryonic resonance production and their **hypothetic decay  $N^*$  to  $\phi$  mesons**



Modelling of **in-medium effects** in off-shell PHSD:

□  $\phi$  mesons:

- **collisional broadening** of the  $\phi$  spectral function [D. Cabrera et al., PRC 95 (2017) 015201]
- + **multi-step mB and mY reactions** from SU(6) chiral Lagrangian

For  $S=0, I_3=1/2, i, j=$

$$\eta N, K\Lambda, K\Sigma, \rho N, K\Sigma^*, \rho\Delta, \\ K^*\Lambda, K^*\Sigma, K^*\Sigma^* \rightarrow \phi N,$$

For  $S=0, I_3=3/2, i, j=$

$$K\Sigma, \rho N, \eta\Delta, K\Sigma^*, \rho\Delta, \\ K^*\Sigma, K^*\Sigma^* \rightarrow \phi\Delta$$

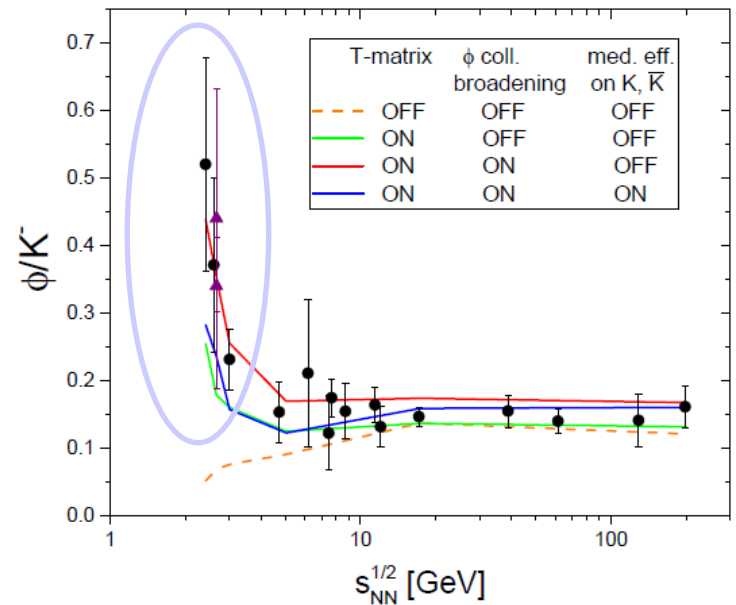
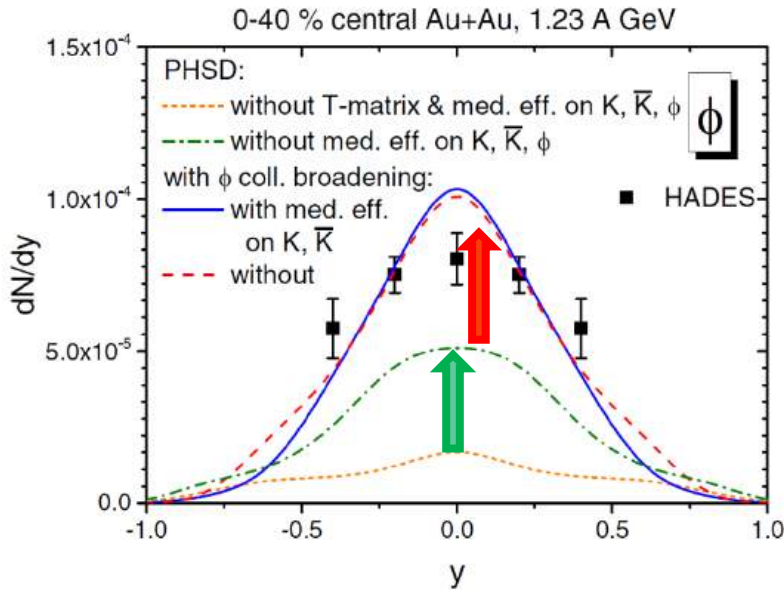
including in-medium effects for

- **antikaon** – improved **G-matrix** from SU(3) mB chiral Lagrangian [D. Cabrera et al., PRC 90 (2014) 055207]
- **kaons** - repulsive nuclear potential

PHSD: T. Song et al., PRC 106, 024903 (2022); PRC 103, 044901 (2021)

UrQMD: H.W. Barz et al., NPA 705 (2002) 223

SMASH: V. Steinberg et al., Phys. Rev. C 99 (2019) 064908 (2019)



**Orange:** without T-matrix, without coll. broadening  $\rightarrow$  underestimate the ratio at low energies

**Green:** with T-matrix, but without coll. broadening  $\rightarrow$  enhance the ratio

**Red:** with T-matrix + coll. broadening but without med. eff. on  $K, Kbar$   $\rightarrow$  enhance the ratio

**Blue:** all included, i.e. with T-matrix + coll. broadening + med. eff. on  $K, Kbar$   $\rightarrow$  suppress the ratio due to the too strong enhanced  $K^-$  production at low energy (compared to the HADES data)

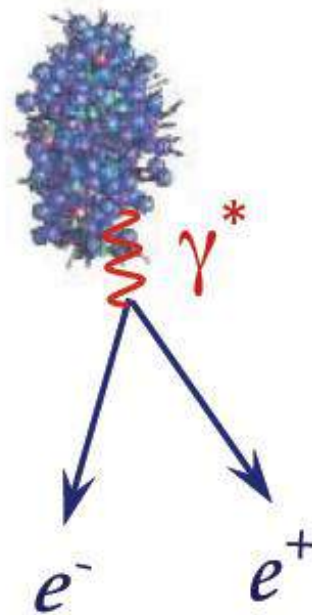
Can be clarified by **CBM**:

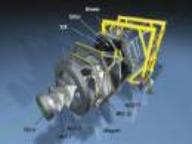
The “enhanced”  $\phi$  multiplicity and  $\phi/K^-$  ratio close to threshold can be understood:

**PHSD:** by incorporation of in-medium effects + novel mB production channels

**UrQMD, SMASH:** by assuming the hypothetical decay  $N^*$  to  $\phi$  mesons

# Electromagnetic probes of the strongly interaction matter: dileptons

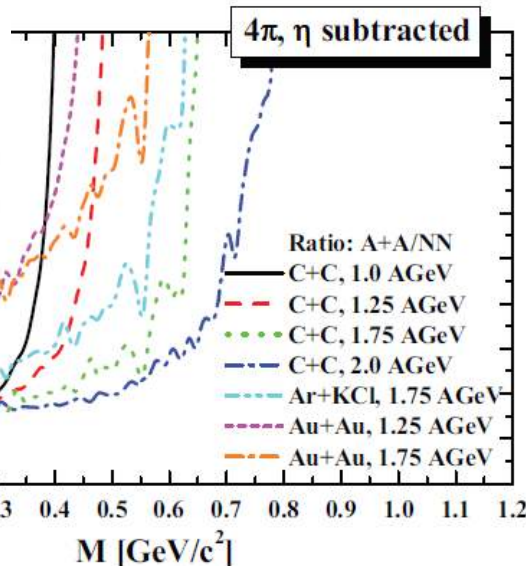
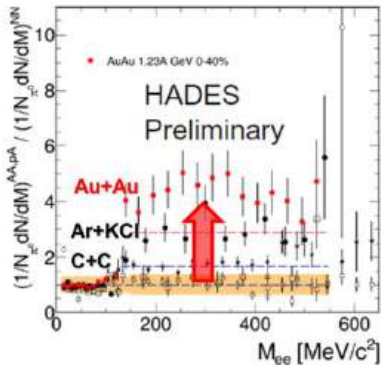
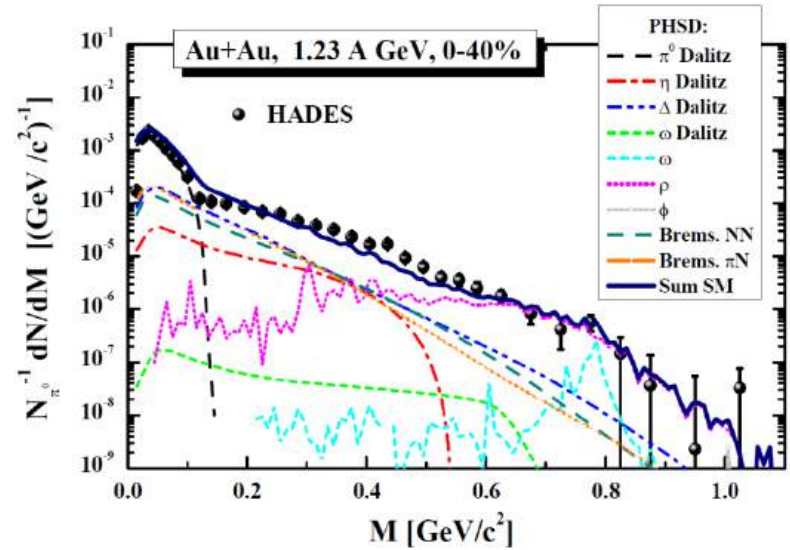
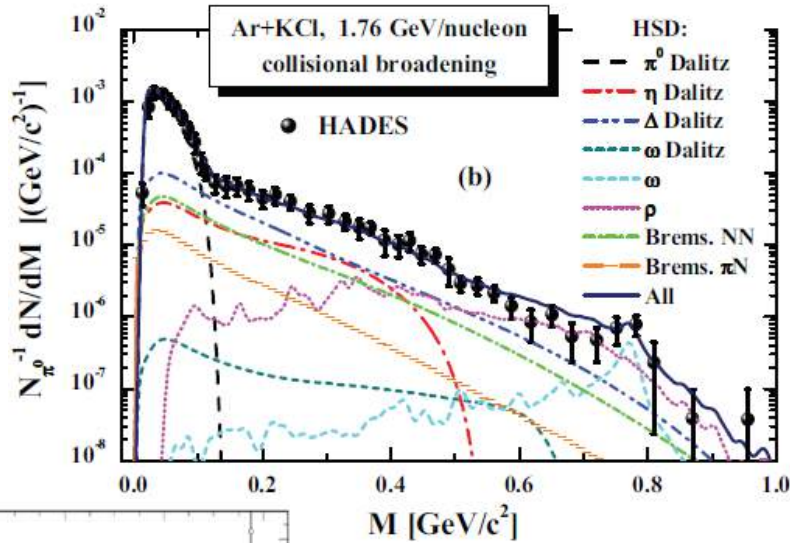




# Dileptons at SIS energies - HADES

E. B., J. Aichelin, M. Thomere, S. Vogel, M. Bleicher, PRC 87 (2013) 064907

I. Schmidt, E.B., M. Gumberidze, R. Holzmann, PRD 104 (2021), 015008



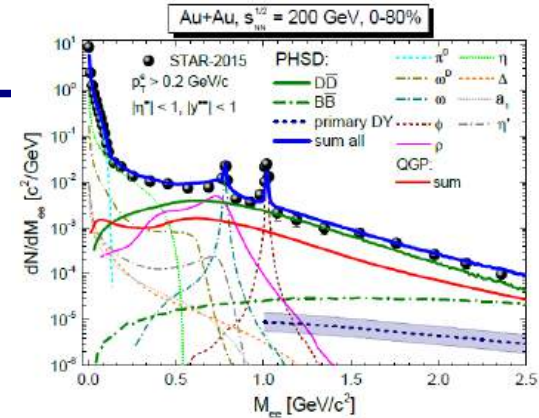
- **Strong in-medium enhancement** of dilepton yield in Au+Au vs. NN
- **Increases with the system size:**

- 1) **multiple  $\Delta$  regeneration** – dilepton emission from intermediate  $\Delta$ 's which are part of the reaction cycles  $\Delta \rightarrow \pi N$ ;  $\pi N \rightarrow \Delta$  and  $NN \rightarrow N\Delta$ ;  $N\Delta \rightarrow NN$
- 2) **pN bremsstrahlung** which scales with  $N_{\text{bin}}$  and not with  $N_{\text{part}}$ , i.e. pions
- 3) **Collisional broadening** of  $\rho, \omega, \phi$  mesons

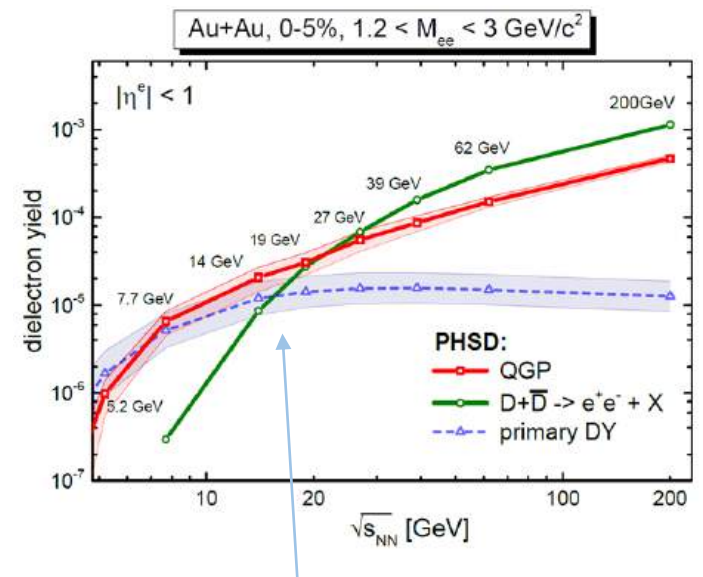
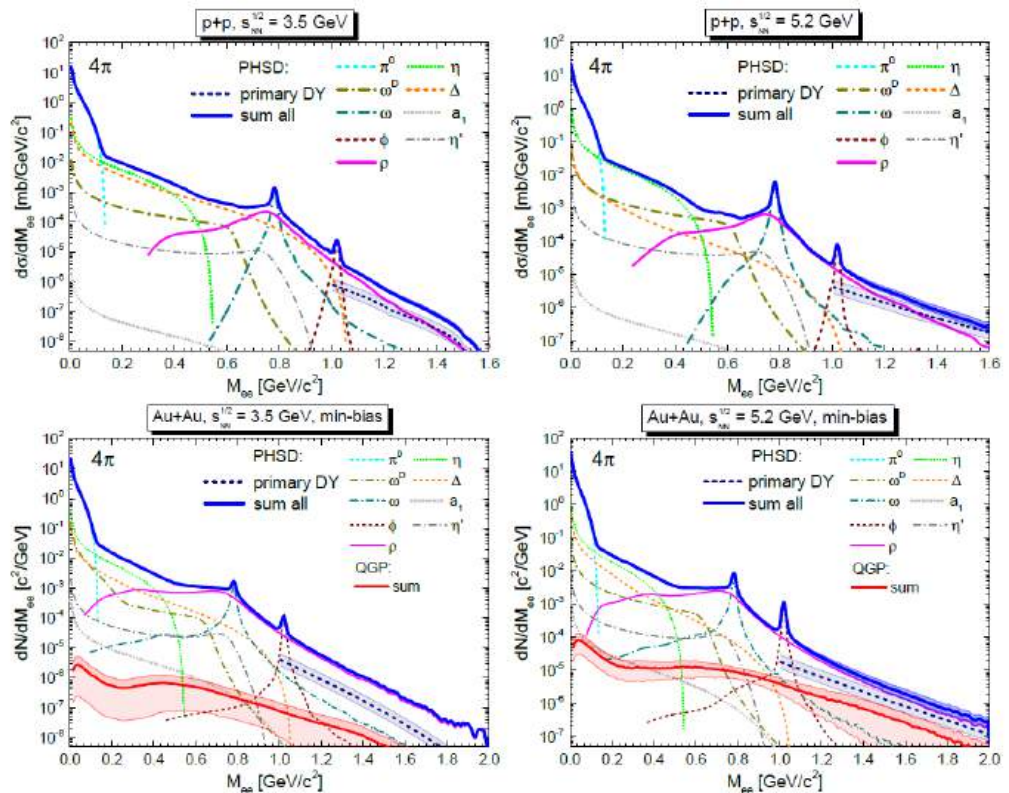
# Dileptons: predictions for CBM

PHSD dilepton spectra including a collisional broadening of the **vector meson** spectral functions + primary DY + QGP + correlated charm

$q\bar{q} \rightarrow e^+e^-$   
 $q\bar{q} \rightarrow ge^+e^-$   
 $gq(\bar{q}) \rightarrow q(\bar{q})e^+e^-$



Excitation function of dilepton yield integrated for  $1.2 < M < 3$  GeV



- ☐ Dileptons from **QGP** **overshine** **charm** dileptons with decreasing beam energy
- ☐ **Primary DY** could be “subtracted” from AA dilepton spectra using pp data

➔ **Good perspectives for CBM!**

Adrian W. R. Jorge & Taesoo Song, in progress

# QMD dynamics, cluster production



**PHQMD:**

J. Aichelin et al., PRC 101 (2020) 044905;  
S. Gläsel et al., PRC 105 (2022) 1;  
V. Kireyeu et al., PRC 105 (2022) 044909;  
G. Coci et al., PRC 108 (2023) 1, 014902;  
V. Kireyeu et al., arXiv:2411.04969

# Cluster recognition: Minimum Spanning Tree (MST)

R. K. Puri, J. Aichelin, J.Comp. Phys. 162 (2000) 245-266

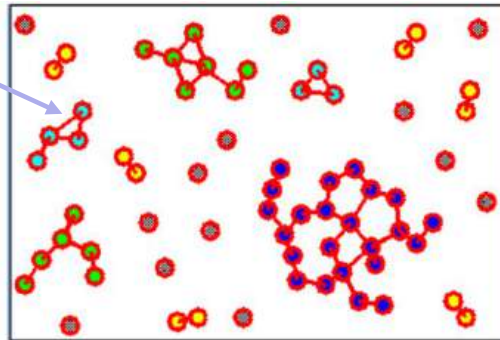
The **Minimum Spanning Tree (MST)** is a **cluster recognition** method applicable for the (asymptotic) **final states** where coordinate space correlations may only survive for bound states.

The MST algorithm searches for **accumulations of particles in coordinate space**:

1. Two particles are 'bound' if their **distance in the cluster rest frame** fulfills

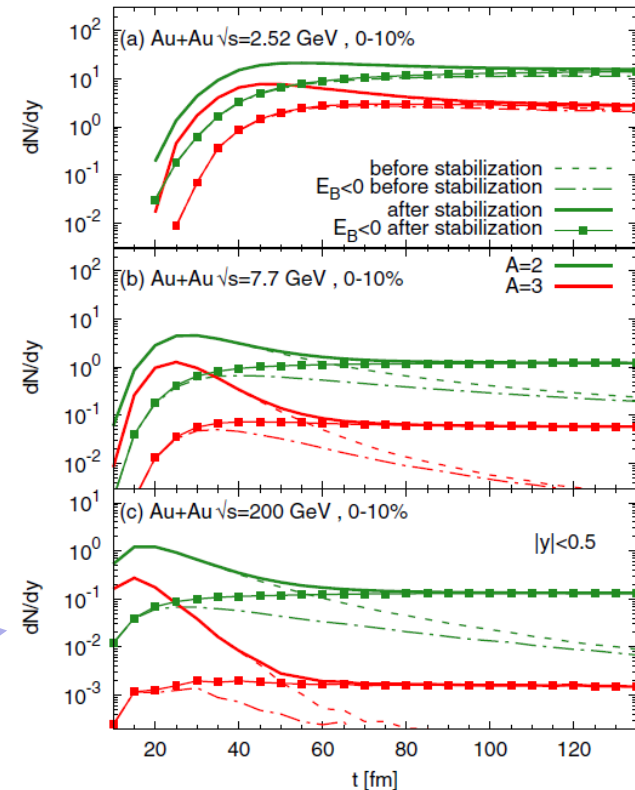
$$|\vec{r}_i - \vec{r}_j| \leq 4 \text{ fm} \quad (\text{range of NN potential})$$

2. Particle is **bound to a cluster** if it **binds with at least one particle of the cluster**



## Advanced MST (aMST)

- ❑ **MST + extra condition:  $E_B < 0$**   
**negative binding energy** for identified clusters
- ❑ **Stabilization procedure** – to correct artifacts of the semi-classical QMD:  
recombine the final “lost” nucleons back into clusters if they left the cluster without rescattering



## “Kinetic mechanism”

- 1) hadronic inelastic reactions  $NN \leftrightarrow d\pi$ ,  $\pi NN \leftrightarrow d\pi$ ,  $NNN \leftrightarrow dN$
- 2) hadronic elastic  $\pi+d$ ,  $N+d$  reactions

Transition rate for  $3 \rightarrow 2$  reactions:

$$\frac{\Delta N_{coll}[3 + 4 + 5 \rightarrow 1(d) + 2]}{\Delta N_3 \Delta N_4 \Delta N_5} = P_{3,2}(\sqrt{s})$$

$$P_{3,2}(\sqrt{s}) = F_{spin} F_{iso} P_{2,3}(\sqrt{s}) \frac{E_1^f E_2^f}{2E_3 E_4 E_5} \frac{R_2(\sqrt{s}, m_1, m_2)}{R_3(\sqrt{s}, m_3, m_4, m_5)} \frac{1}{\Delta V_{cell}}$$

Energy and momentum of final particles

2,3-body phase space integrals [Byckling, Kajantie]

W. Cassing, NPA 700 (2002) 618

$$P_{2,3}(\sqrt{s}) = \sigma_{tot}^{2,3}(\sqrt{s}) v_{rel} \frac{\Delta t}{\Delta V_{cell}}$$

→ solved by **stochastic method**

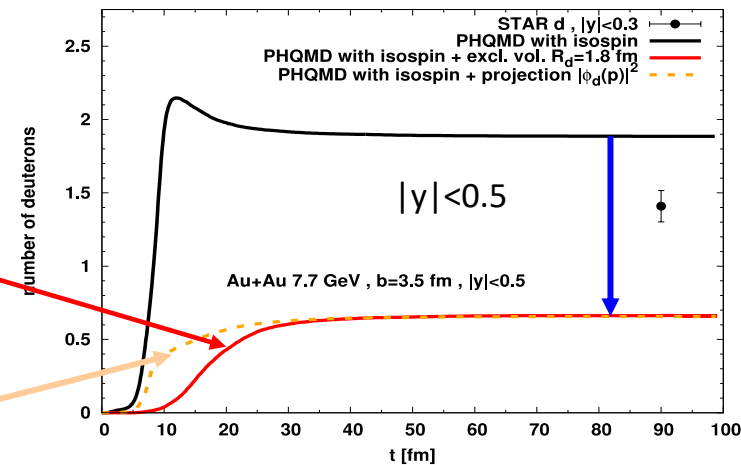
+ inclusion of **all possible isospin channels** which enhance d production

+ accounting of **quantum properties of d**, modelled by:

- 1) the **finite-size excluded volume effect** in coordinate space

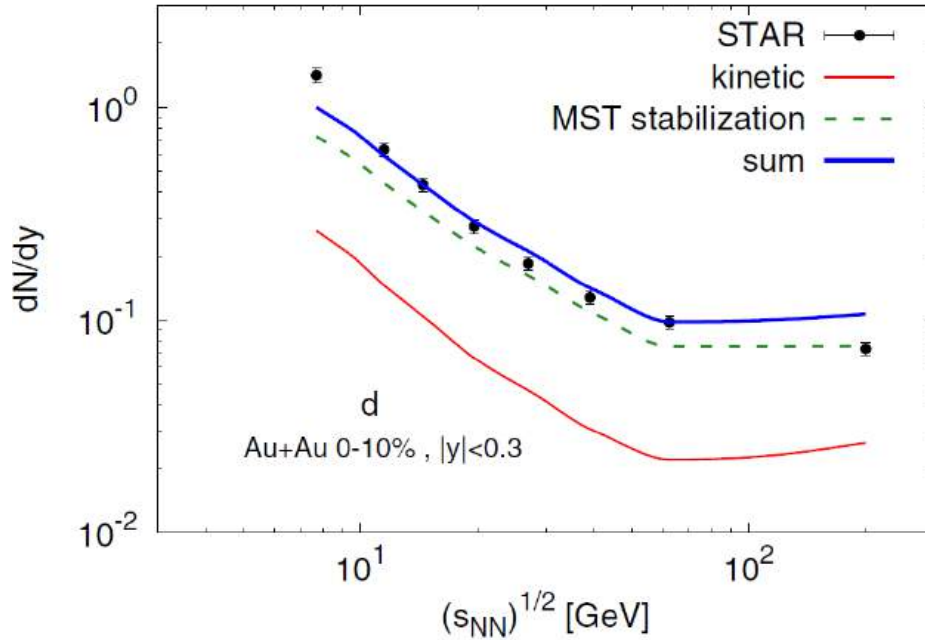
$$|\vec{r}(i)^* - \vec{r}(d)^*| < R_d$$

- 2) the **momentum correlations of p and n** inside d by and projection of relative momentum of p+n pair on d wave-function in momentum space  $|\phi_d(p)|^2$  which lead to a **strong reduction** of d production



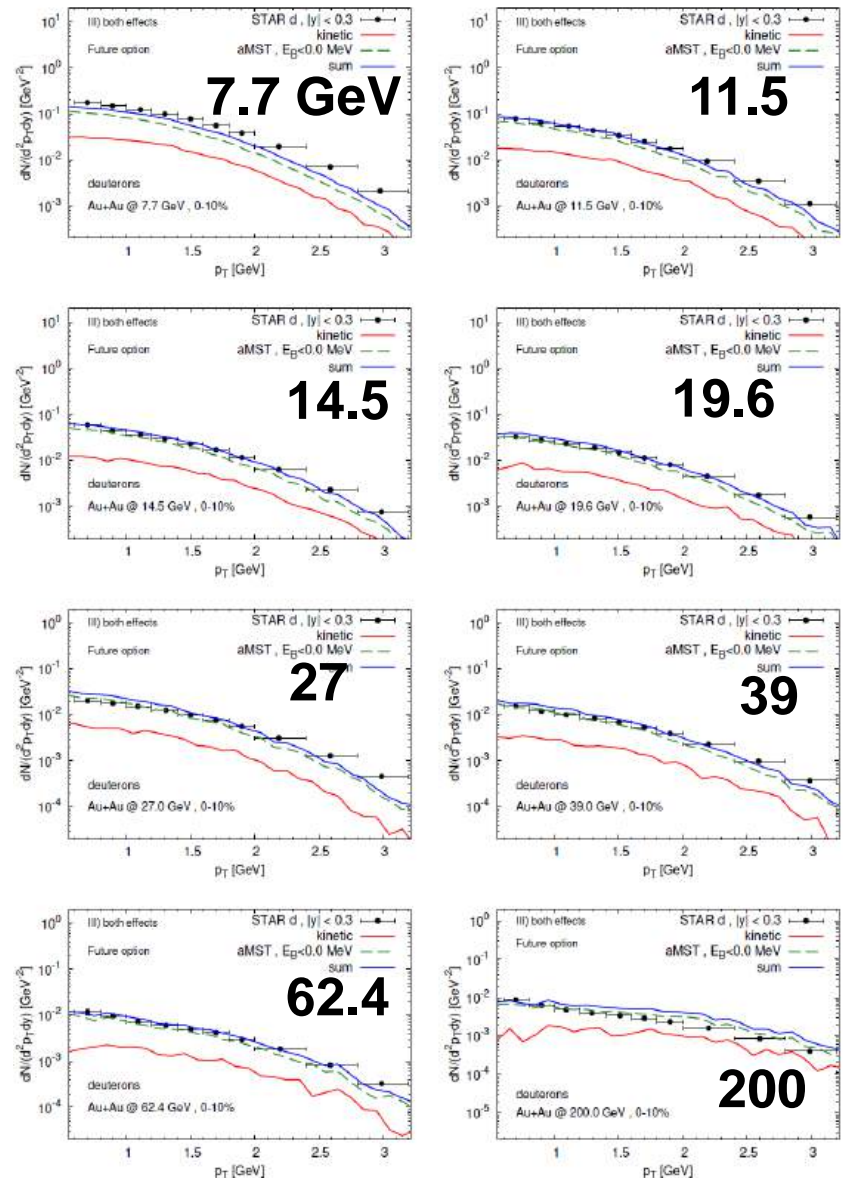
# Kinetic vs. potential deuteron production

Excitation function  $dN/dy$  of deuterons at midrapidity



- ❑ PHQMD provides a good description of STAR data
- ❑ Functional forms of  $y$ - and  $p_T$ -spectra are slightly different for kinetic and potential deuterons
- ❑ **The potential mechanism is dominant for d production at all energies!**

$p_T$  – spectra (BES RHIC)



## How to learn about EoS from clusters:

→ spectra and  $v_1$ ,  $v_2$  of light clusters  
with different EoS in PHQMD:  
hard, soft, momentum dependent potential



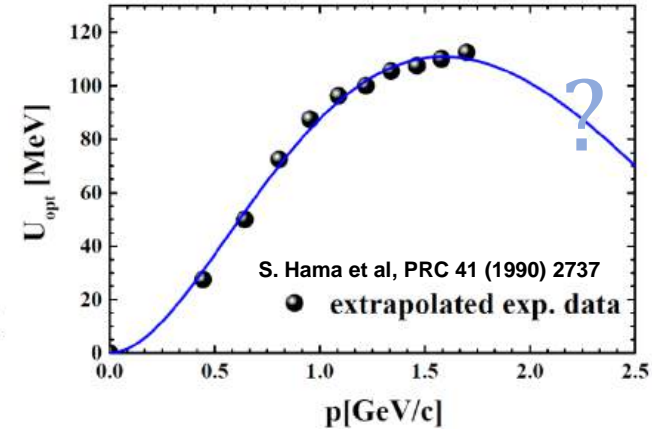
## 2) Momentum dependent potential :

$$V(\mathbf{r}_1, \mathbf{r}_2, \mathbf{p}_{01}, \mathbf{p}_{02}) = (a\Delta p + b\Delta p^2) \exp[-c\sqrt{\Delta p}] \delta(\mathbf{r}_1 - \mathbf{r}_2)$$

$$\Delta p = \sqrt{(\mathbf{p}_{01} - \mathbf{p}_{02})^2}$$

Parameters **a, b, c** are fitted to the "optical" potential (Schrödinger equivalent potential  $U_{SEP}$ )

extracted from elastic scattering data in pA:  $U_{SEQ}(p) = \frac{\int^{p_F} V(\mathbf{p} - \mathbf{p}_1) dp_1^3}{\frac{4}{3}\pi p_F^3}$

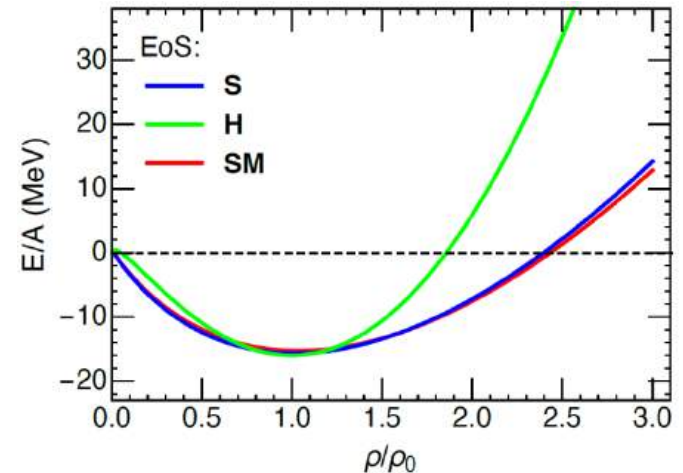


❖ In infinite matter a potential corresponds to an EoS:

$$E/A(\rho) = \frac{3}{5}E_F + V_{Skyrme\ stat}(\rho) + V_{mom}(\rho)$$

$$V_{Skyrme} = \alpha \frac{\rho}{\rho_0} + \beta \frac{\rho^\gamma}{\rho_0}$$

EoS for infinite cold nuclear matter at rest



compression modulus **K** of nuclear matter:

$$K = -V \frac{dP}{dV} = 9\rho^2 \frac{\partial^2(E/A(\rho))}{(\partial\rho)^2} \Big|_{\rho=\rho_0}$$

| E.o.S.   | $\alpha$ [MeV] | $\beta$ [MeV] | $\gamma$ | K [MeV] |
|--|----------------|---------------|----------|---------|
| S  | -383.5         | 329.5         | 1.15     | 200     |
| H  | -125.3         | 71.0          | 2.0      | 380     |
| SM   | -478.87        | 413.76        | 1.10     | 200     |
| a [MeV <sup>-1</sup> ] b [MeV <sup>-2</sup> ] c [MeV <sup>-1</sup> ] |                |               |          |         |
| 236.326 -20.73 0.901   |                |               |          |         |

→ **H,S,SM potentials** (forces) act differently on different **observables**:

1) **yield  $dN/dy$  at midrapidity:**

protons:  $SM = S < H$

deuterons:  $SM = S > H$

2) **directed flow  $v_1$ :**

protons:  $SM \geq H > S$

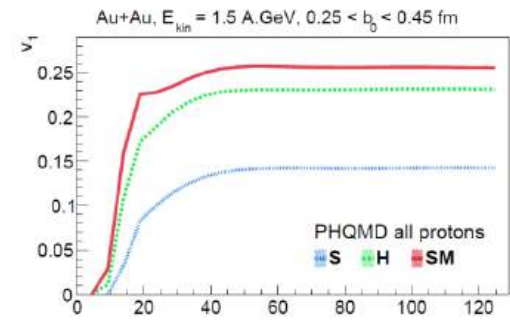
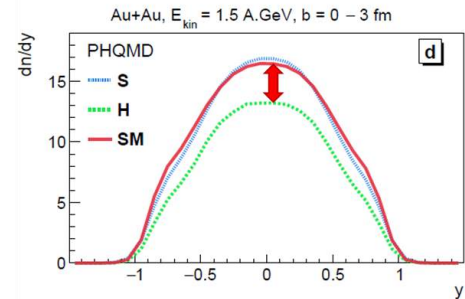
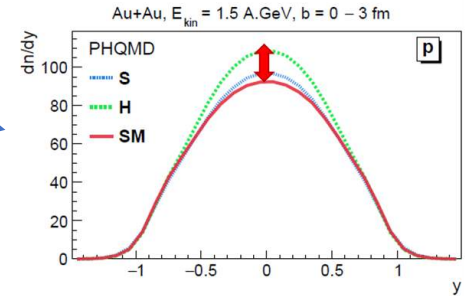
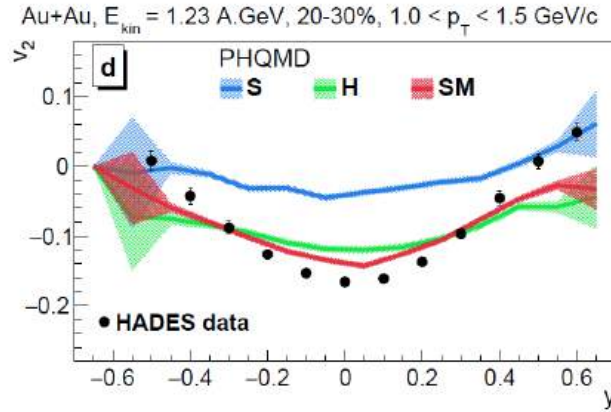
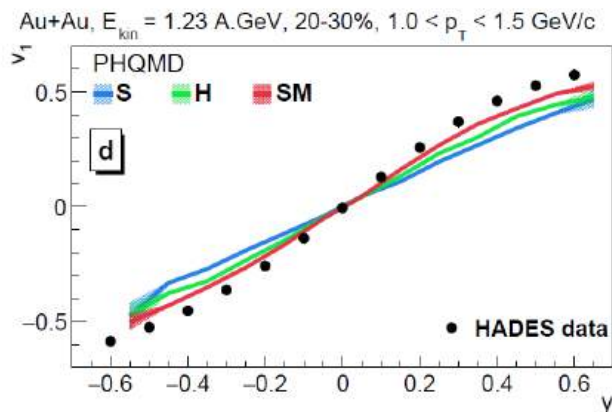
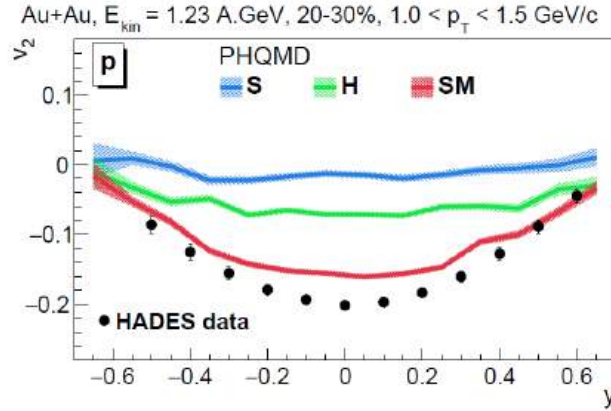
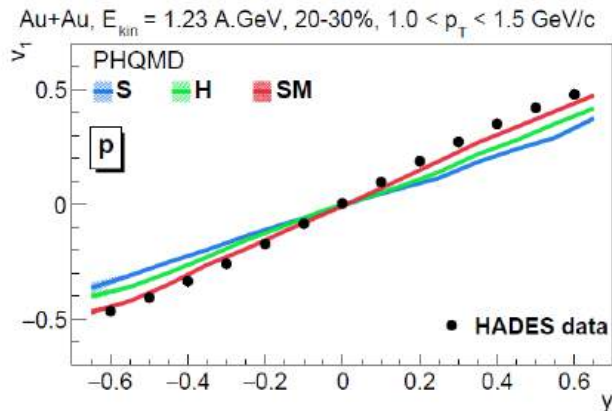
deuterons:  $SM \geq H > S$

3) **elliptic flow  $|v_2|$ :**

protons:  $SM > H > S$

deuterons:  $SM \geq H > S$

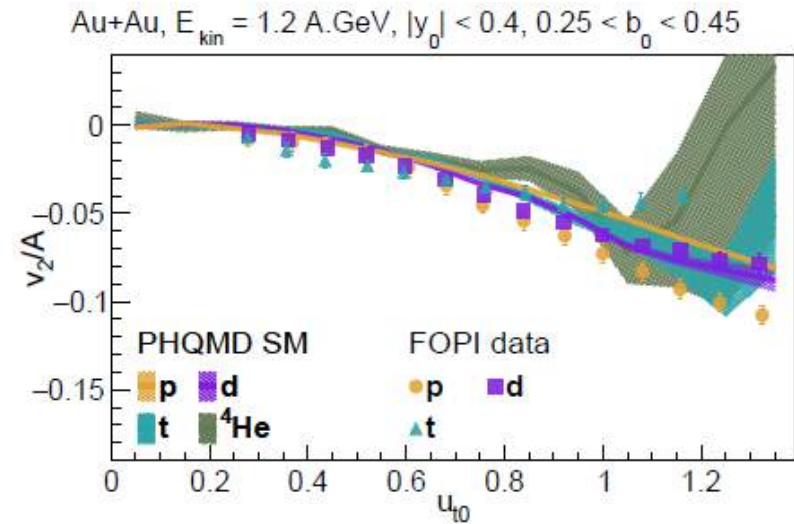
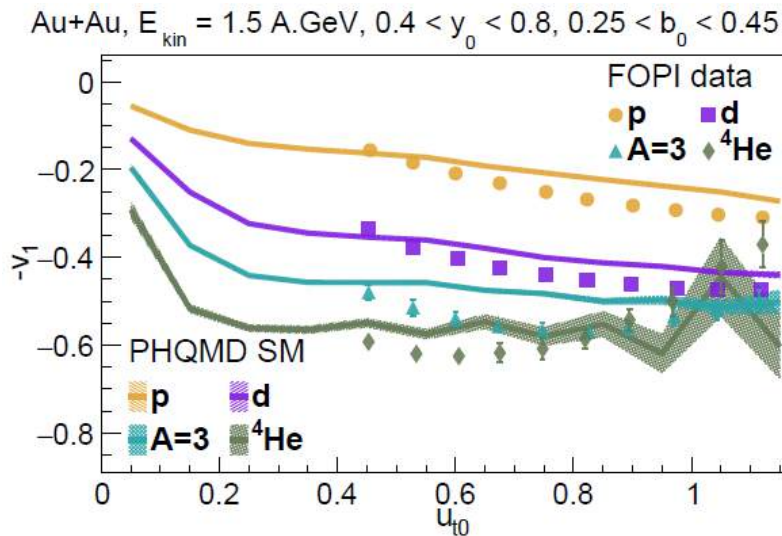
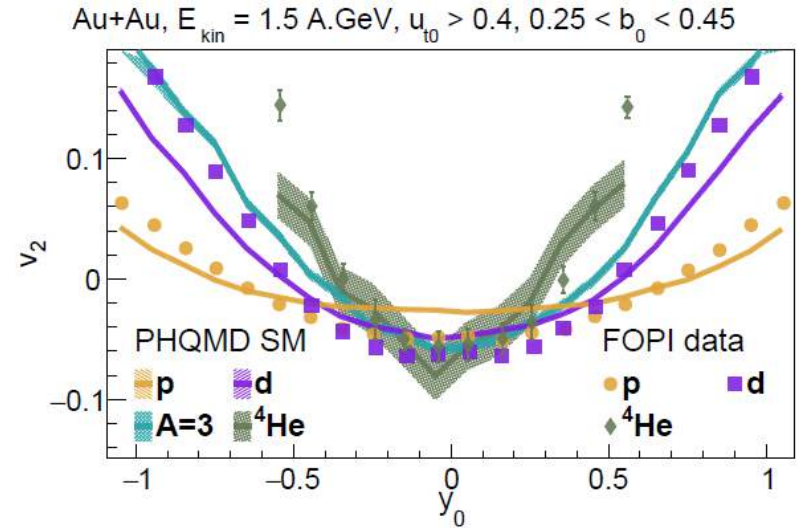
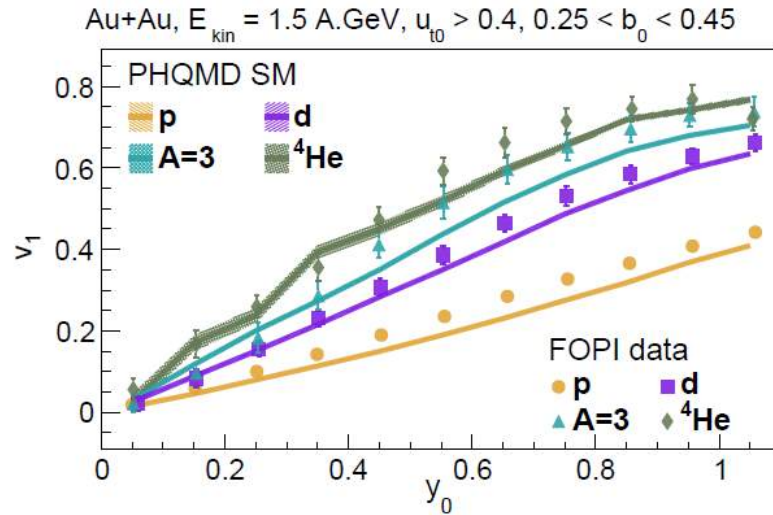
V. Kireyeu et al., arXiv:2411.04969



**Flow  $v_1$  with SM develops earlier than for H EoS and earlier than for S EoS**

\*Cf. earlier studies (80<sup>th</sup>, 90<sup>th</sup>, ...) on p flow by J. Aichelin, W. Cassing, P. Danielewicz, C. Fuchs, C.-M. Ko, U. Mosel, H. Stoecker, ....

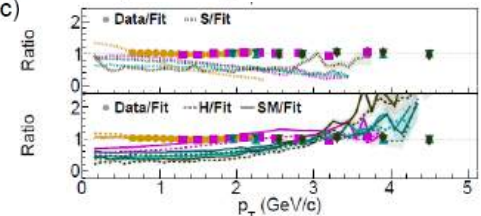
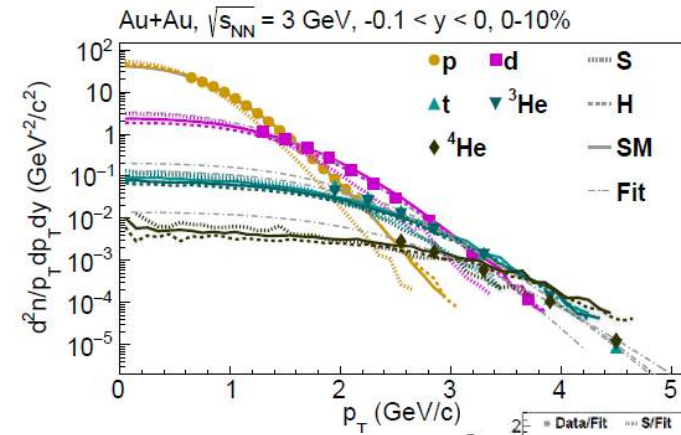
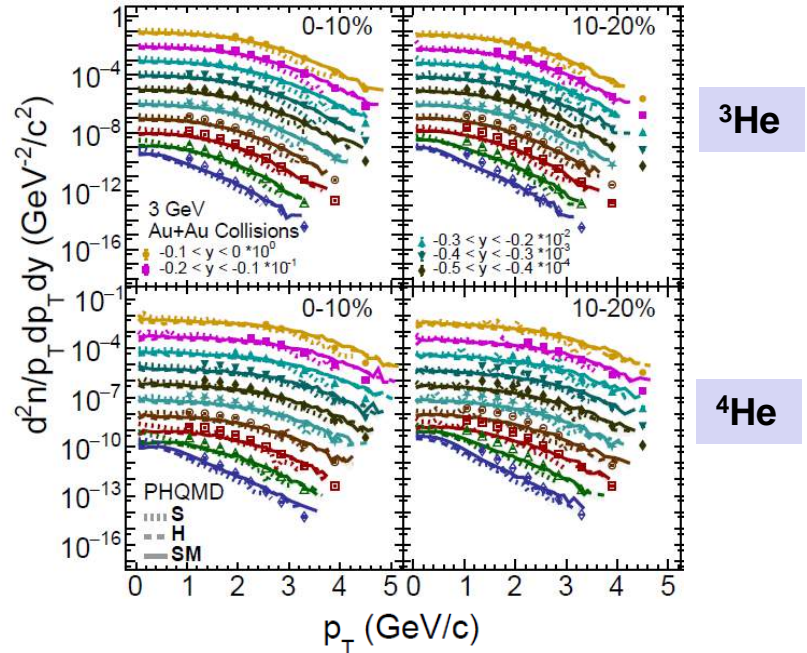
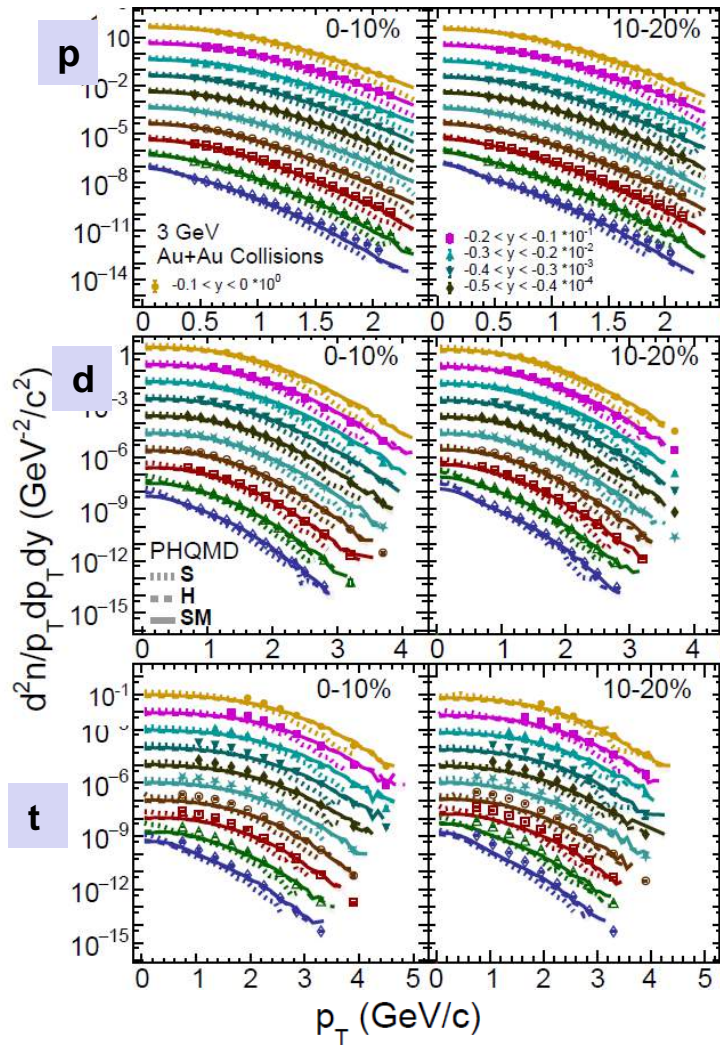
# EoS dependence of flow at SIS: FOPI



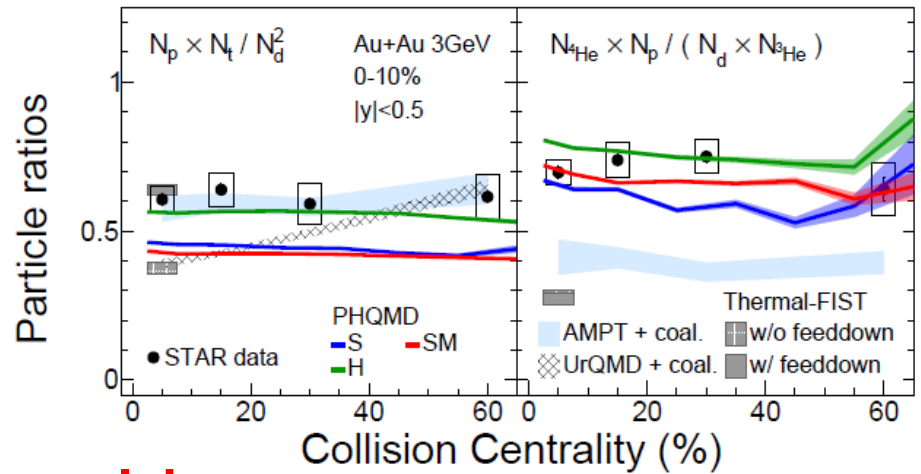
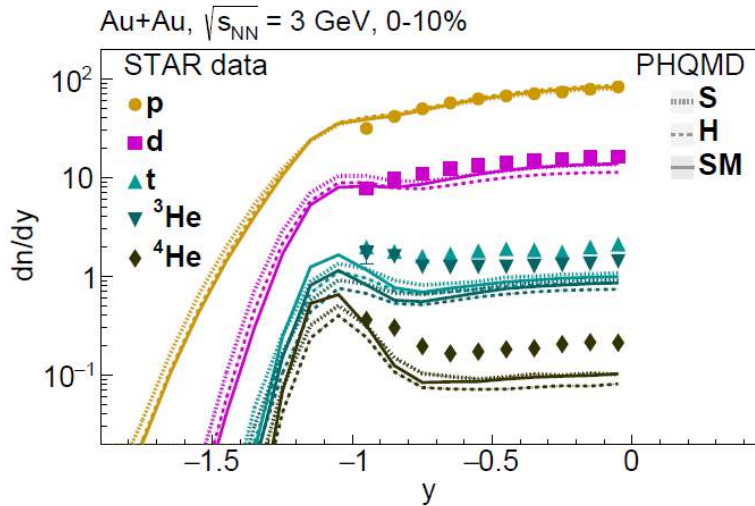
- Strong EoS dependence of  $v_1(y)$ ,  $v_2(y)$  and  $v_2(p_T)$  of clusters
- FOPI and HADES data favor a soft momentum dependent potential



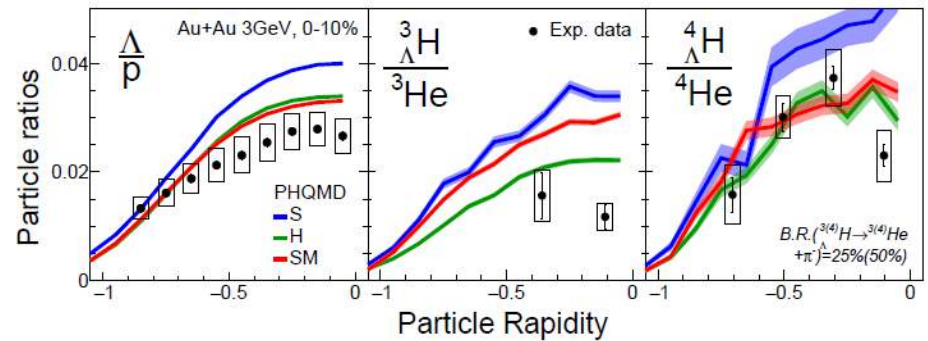
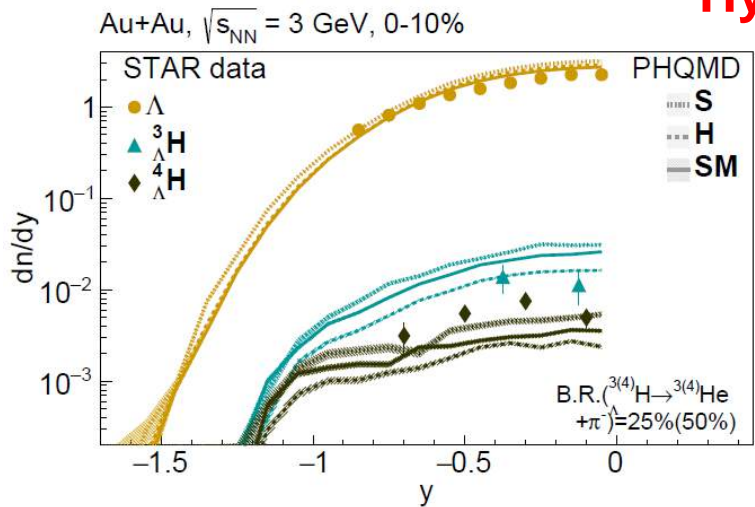
# EoS dependence of $p_T$ -spectra at STAR : $s^{1/2}=3$ GeV



STAR  $p_T$  data favor a hard or soft-momentum dependent potential (H/SM)

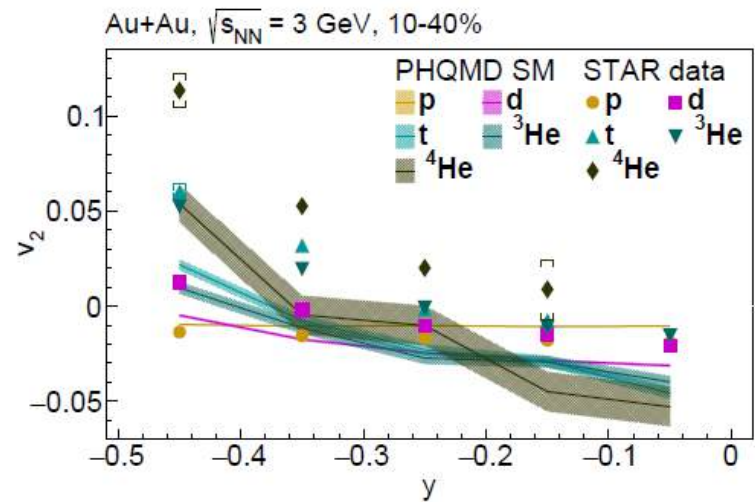
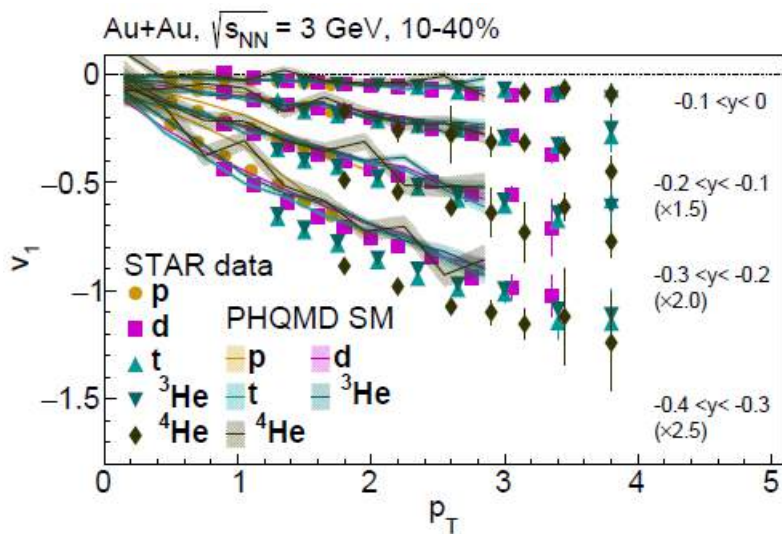
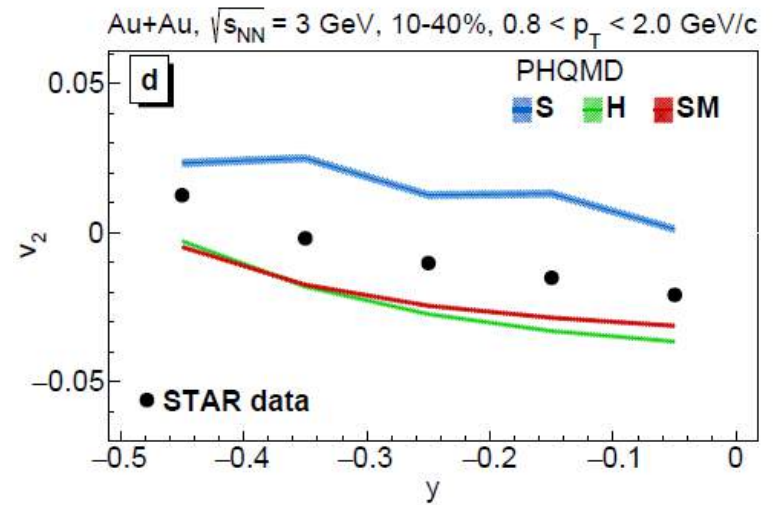
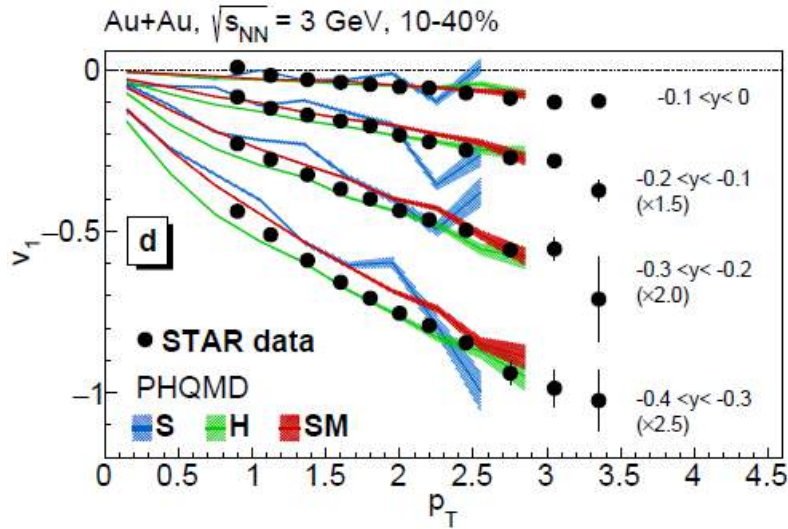


## Hypernuclei:

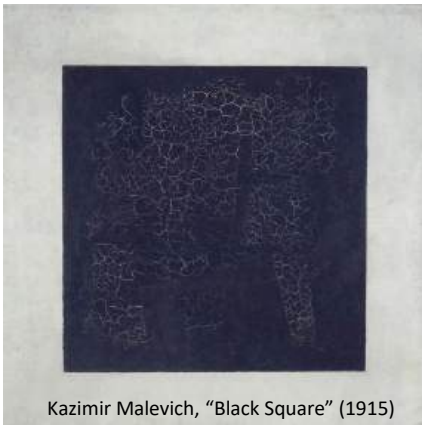


☐ STAR data on  $dn/dy$  favor a hard or soft-momentum dependent potential (H/SM)

# EoS dependence of flow at STAR : $s^{1/2}=3$ GeV



**STAR data on  $v_1, v_2$  favor a hard or soft-momentum dependent potential (H/SM)**



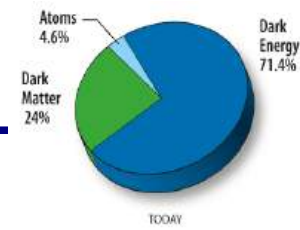
Kazimir Malevich, "Black Square" (1915)

## Physics beyond the standard model (SM): search for dark matter with dileptons

DM ,candidate'

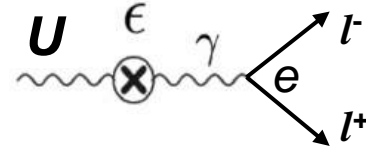


# Light dark photons searches with p+p, p+A



The '**vector**' portal : existence of a **U(1)-U(1)'** gauge symmetry group mixing

$$\mathcal{L}_{A'} = -\frac{1}{4}F'^{\mu\nu}F'_{\mu\nu} + \frac{1}{2} \frac{\epsilon}{\cos\theta_W} B^{\mu\nu}F'_{\mu\nu} - \frac{1}{2}m_{A'}^2 A'^{\mu}A'_{\mu}$$



Notation for 'dark photon': A' or U- boson

**Unknown:** kinetic mixing parameter  $\epsilon$  and mass  $M_U$

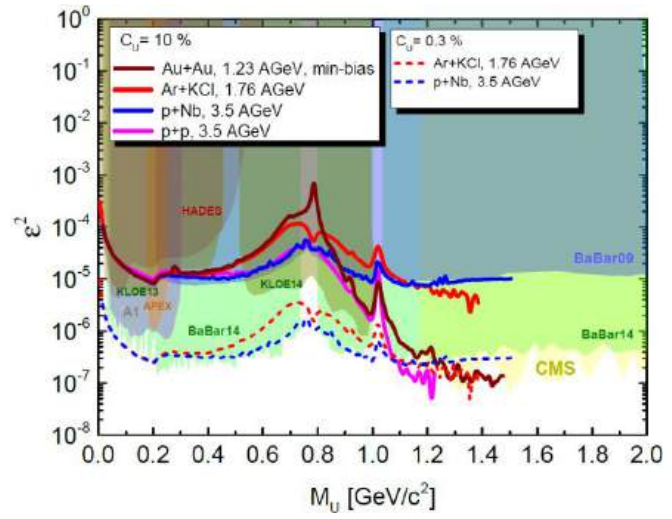
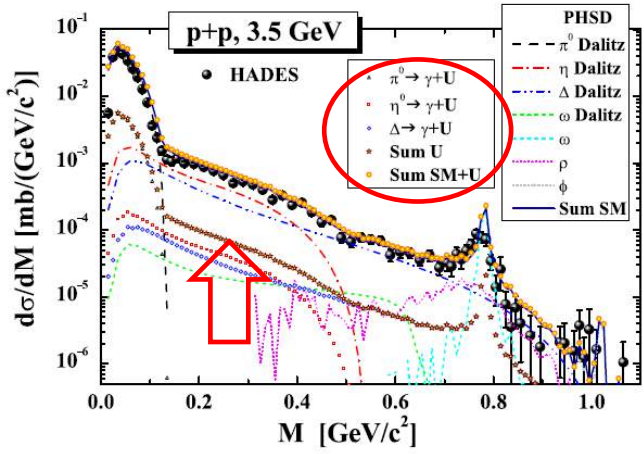


- $\pi^0 \rightarrow \gamma + U$ ,
- $\eta \rightarrow \gamma + U, U \rightarrow e^+e^-$
- $\Delta \rightarrow N + U$
- $K^+ \rightarrow \pi^+ + U$
- $V \rightarrow U, \dots$

B. Holdom, PL B 166, 196 (1986)  
B. Batell et al., PRD 80, 095024 (2009)



The **upper limit for the kinetic mixing parameter  $\epsilon^2(M_U)$**  of light dark photons extracted from the **PHSD dilepton spectra** - with 10 and 3% allowed surplus of the total SM yield by an additional **DM yield** at given M:



HADES: G. Agakishiev et al., Phys. Lett. B 731, 265 (2014)

PHSD: I. Schmidt, E.B., M. Gumberidze, R. Holzmann, Phys.Rev.D 104 (2021) 015008

PHSD: Adrian W. Romero Jorge, 2409.20141, 2412.02536

# Summary



- ❑ **FAIR** is an excellent facility to study the properties of the strongly interacting matter at high density and finite temperature
- ❑ **Transport theory** is the general basis for an understanding of nuclear dynamics on a microscopic level

## Perspectives with CBM:

- study of **EoS by clusters**,
- properties of **QGP at high  $\mu_B$**  (in small volume – “droplets”) by dileptons,
- possible 1<sup>st</sup> order phase transition,
- **chiral symmetry restorations** by dileptons and strangeness,
- subthreshold **charm** production,
- elementary reactions **p+p,  $\pi$ +p and p+A**
  
- search for dark matter (dark photons, axions) via dileptons (and missing tracks – using AI ?)

**Outlook** (challenges for theory):

*PHSD/PHQMD is prepared for that!*

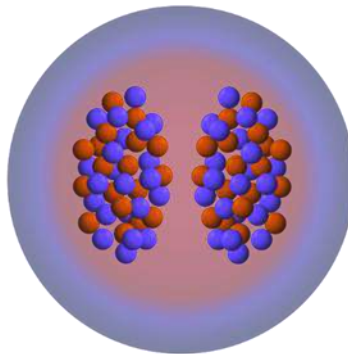
- More precise EoS at large  $\mu_B$
- Possible 1<sup>st</sup> order phase transition at large  $\mu_B$ ?!



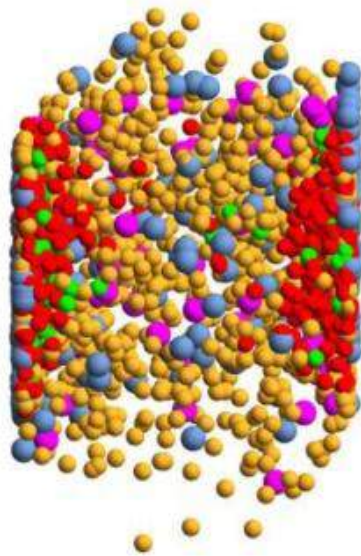
**Thank you for your  
attention!**

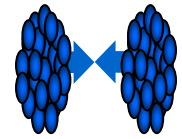
**Thanks to the PHSD/PHQMD  
team!**

**Backup slides:**



# Development of the microscopic transport theory: from BUU to Kadanoff-Baym dynamics





# History: semi-classical BUU equation



Ludwig Boltzmann

**Boltzmann-Uehling-Uhlenbeck equation** (non-relativistic formulation)  
 - propagation of particles in the **self-generated Hartree-Fock mean-field potential**  $U(r,t)$  with an **on-shell collision term**:

$$\frac{\partial}{\partial t} f(\vec{r}, \vec{p}, t) + \frac{\vec{p}}{m} \vec{\nabla}_{\vec{r}} f(\vec{r}, \vec{p}, t) - \vec{\nabla}_{\vec{r}} U(\vec{r}, t) \vec{\nabla}_{\vec{p}} f(\vec{r}, \vec{p}, t) = \left( \frac{\partial f}{\partial t} \right)_{coll}$$

**collision term:**  
 elastic and inelastic reactions

$f(\vec{r}, \vec{p}, t)$  is the **single particle phase-space distribution function**  
 - probability to find the particle at position  $r$  with momentum  $p$  at time  $t$

□ **self-generated Hartree-Fock mean-field potential:**

$$U(\vec{r}, t) = \frac{1}{(2\pi\hbar)^3} \sum_{\beta_{occ}} \int d^3 r' d^3 p V(\vec{r} - \vec{r}', t) f(\vec{r}', \vec{p}, t) + (Fock \text{ term})$$

□ **Collision term for 1+2→3+4 (let's consider fermions) :**

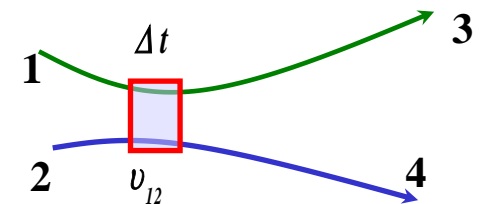
$$I_{coll} = \frac{4}{(2\pi)^3} \int d^3 p_2 d^3 p_3 \int d\Omega |v_{12}| \delta^3(\vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4) \cdot \frac{d\sigma}{d\Omega} (1+2 \rightarrow 3+4) \cdot P$$

**Probability including Pauli blocking of fermions:**

$$P = \underbrace{f_3 f_4 (1 - f_1) (1 - f_2)}_{\text{Gain term}} - \underbrace{f_1 f_2 (1 - f_3) (1 - f_4)}_{\text{Loss term}}$$

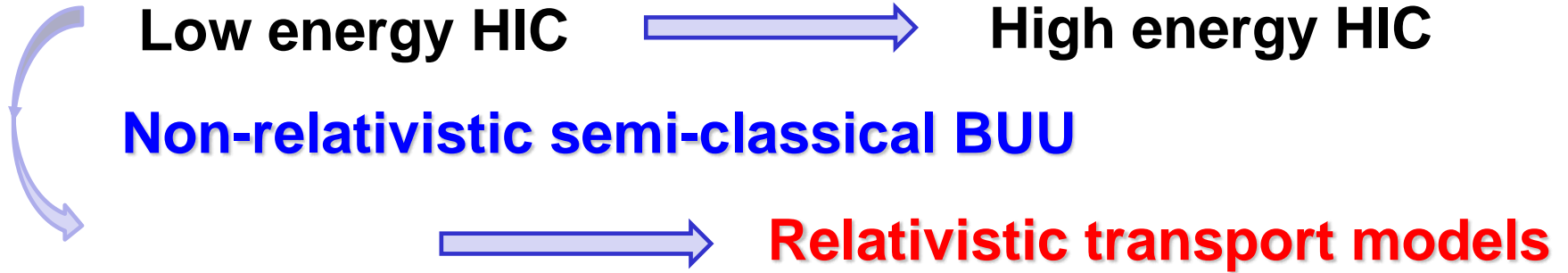
**Gain term: 3+4→1+2**

**Loss term: 1+2→3+4**



# History: developments of relativistic transport models

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‘Numerical simulation of medium energy heavy ion reactions’,  
J. Aichelin and G. Bertsch, Phys.Rev.C 31 (1985) 1730-1738



‘Relativistic Vlasov-Uehling-Uhlenbeck model for heavy-ion collisions’  
Che-Ming Ko, Qi Li, Phys.Rev. C37 (1988) 2270

‘Covariant Boltzmann-Uehling-Uhlenbeck approach for heavy-ion collisions’  
Bernhard Blaettel, Volker Koch, Wolfgang Cassing, Ulrich Mosel, Phys.Rev. C38 (1988) 1767;

‘Relativistic BUU approach with momentum dependent mean fields’  
T. Maruyama, B. Blaettel, W. Cassing, A. Lang, U. Mosel, K. Weber, Phys.Lett. B297 (1992) 228

‘The Relativistic Landau-Vlasov method in heavy ion collisions’  
C. Fuchs, H.H. Wolter, Nucl.Phys. A589 (1995) 732

■ ■ ■

\* Alternative to BUU: **QMD** – non-covariant EoM (contrary to BUU), but not a mean-field!

# Covariant transport equation



## □ Covariant relativistic on-shell BUU equation :

from many-body theory by connected Green functions in phase-space + mean-field limit for the propagation part (VUU)

$$\left\{ \left( \Pi_\mu - \Pi_\nu (\partial_\mu^p U_\nu^\nu) - m^* (\partial_\mu^p U_S^\nu) \right) \partial_x^\mu + \left( \Pi_\nu (\partial_\mu^x U_\nu^\nu) + m^* (\partial_\mu^x U_S^\nu) \right) \partial_p^\mu \right\} f(x, p) = I_{coll}$$

$$I_{coll} \equiv \sum_{2,3,4} \int d2 d3 d4 [G^+ G]_{1+2 \rightarrow 3+4} \delta^4(\Pi + \Pi_2 - \Pi_3 - \Pi_4)$$

$$d2 \equiv \frac{d^3 p_2}{E_2}$$

$$\times \{ f(x, p_3) f(x, p_4) (1 - f(x, p)) (1 - f(x, p_2))$$

Gain term  
3+4 → 1+2

$$- f(x, p) f(x, p_2) (1 - f(x, p_3)) (1 - f(x, p_4)) \}$$

Loss term  
1+2 → 3+4

where  $\partial_\mu^x \equiv (\partial_t, \vec{\nabla}_r)$

$$m^*(x, p) = m + U_S(x, p) \quad - \text{effective mass}$$

$$\Pi_\mu(x, p) = p_\mu - U_\mu(x, p) \quad - \text{effective momentum}$$

$U_S(x, p)$ ,  $U_\mu(x, p)$  are scalar and vector part of particle self-energies

$\delta(\Pi_\mu \Pi^\mu - m^{*2})$  – mass-shell constraint

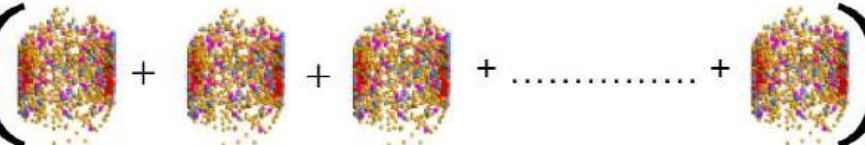
# Numerical solution of BUU EoM

**Testparticle method** or **method of parallel ensembles** :

the 1-body phase space distribution function is described as a sum of  $N$  point-like particles ( $\delta$  –functions).

In the limit of large number of parallel ensembles  $N_t \rightarrow \infty$

$$f(\vec{r}, \vec{p}, t) = \frac{1}{N_t} \sum_{i=1}^{N \cdot N_t} \delta(\vec{r} - \vec{r}_i(t)) \delta(\vec{p} - \vec{p}_i(t)) \quad \text{is a solution of Vlasov EoM}$$

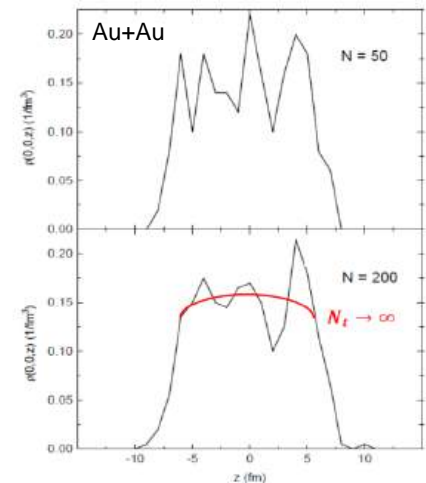
$$\frac{1}{N_t} \left( \begin{array}{c} \text{1} \\ \text{2} \\ \text{3} \\ \dots \\ \text{N} \end{array} \right)$$


- **Testparticle method** provides a smooth density distribution for calculation of **mean-field potential** for particle propagation.
  - No exchange of particles between the parallel ensembles, particles collide only inside one ensemble

➔ **Propagation of test-particles** in time following 'classical' EoM:

$$\dot{\vec{r}}_i = \frac{d\vec{r}_i}{dt} = \frac{\vec{p}_i}{m_i}$$

$$\dot{\vec{p}}_i = \frac{d\vec{p}_i}{dt} = -\vec{\nabla}_{\vec{r}_i} U(\vec{r}_i, t)$$



# Dynamical transport model: collision terms

□ BUU eq. for **different particles of type  $i=1, \dots, n$**

$$Df_i \equiv \frac{d}{dt} f_i = I_{coll} [f_1, f_2, \dots, f_n]$$

Drift term=Vlasov eq.      collision term

$i$  : *Baryons* :  $p, n, \Delta(1232), N(1440), N(1535), \dots, \Lambda, \Sigma, \Sigma^*, \Xi, \Omega; \Lambda_c$

*Mesons* :  $\pi, \eta, K, \bar{K}, \rho, \omega, K^*, \eta', \phi, a_1, \dots, D, \bar{D}, J / \Psi, \Psi', \dots$

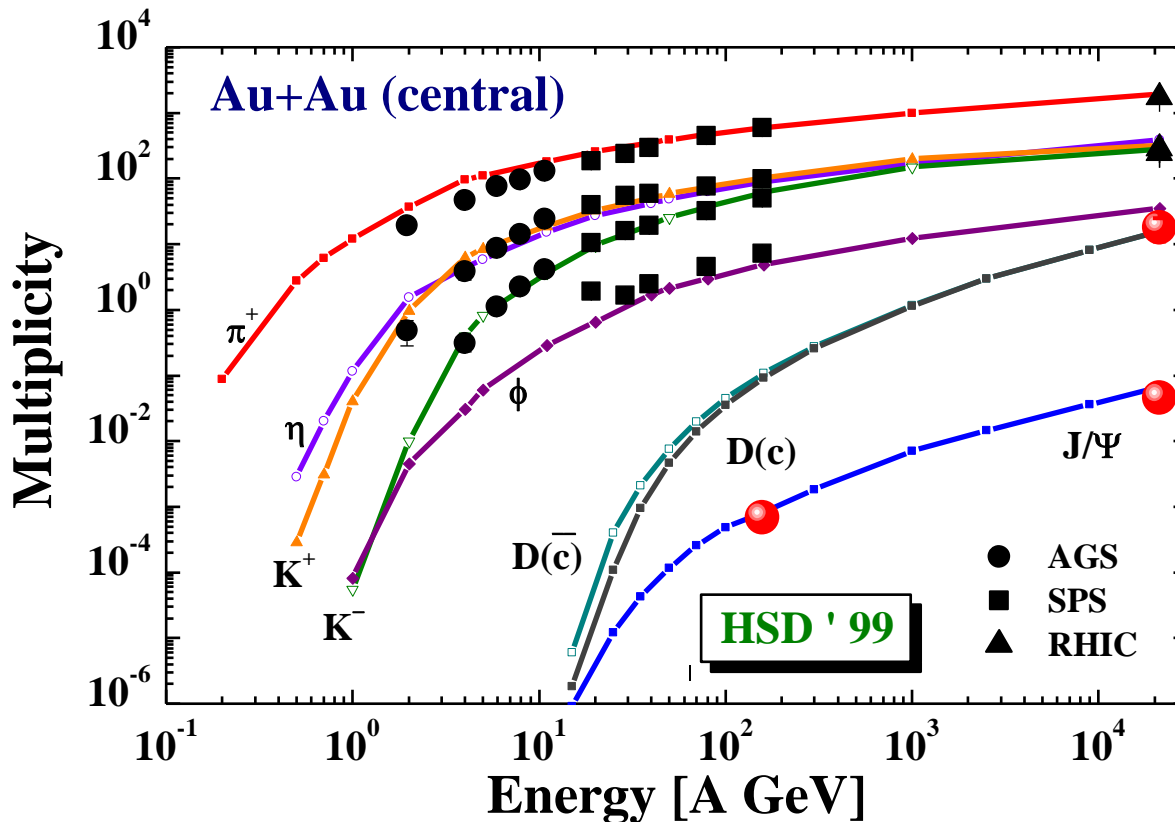
→ **coupled set of BUU equations** for different particles of type  $i=1, \dots, n$

$$\left\{ \begin{array}{l} Df_N = I_{coll} [f_N, f_\Delta, f_{N(1440)}, \dots, f_\pi, f_\rho, \dots] \\ Df_\Delta = I_{coll} [f_N, f_\Delta, f_{N(1440)}, \dots, f_\pi, f_\rho, \dots] \\ \dots \\ Df_\pi = I_{coll} [f_N, f_\Delta, f_{N(1440)}, \dots, f_\pi, f_\rho, \dots] \\ \dots \end{array} \right.$$



# Microscopic transport model for heavy-ion collisions

- very good description of particle production in **pp, pA, pA, AA reactions**
- unique description of nuclear dynamics from **low (~100 MeV) to ultrarelativistic (>20 TeV) energies**



# From weakly to strongly interacting systems

Properties of matter (on hadronic and partonic levels) in heavy-ion collisions:

**QGP** – strongly interacting system! Degrees of freedom – dressed partons

**Hadronic matter** – in-medium effects – modification of hadron properties at finite  $T, \mu_B$  (vector mesons, strange mesons)

Many-body theory:

**Strong interaction** → large width = short life-time

→ broad spectral function → **quantum object**

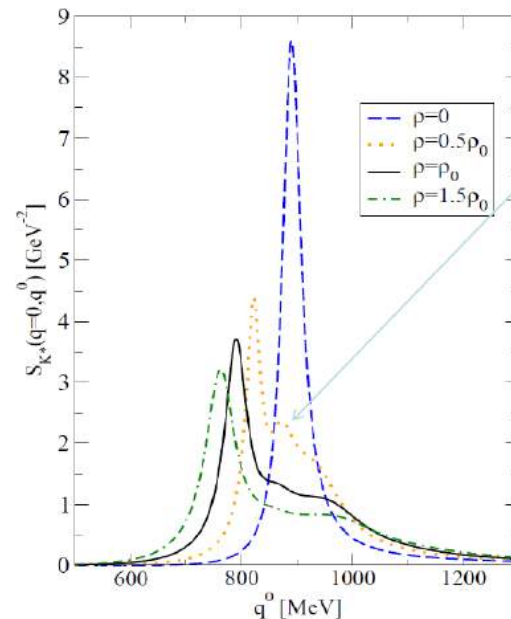
▪ How to describe the dynamics of broad strongly interacting quantum states in transport theory?

☐ go beyond a semi-classical BUU

It is doable with **quantum Kadanoff-Baym equations**

☐ generalized transport equations based on Kadanoff-Baym dynamics

Kbar\* spectral function



$\Lambda(1783)N^{-1}$   
and  
 $\Sigma(1830)N^{-1}$   
excitations

# Dynamical description of strongly interacting systems

Quantum field theory →

**Kadanoff-Baym dynamics** for resummed single-particle Green functions  $S^<$

(1962)

$$\hat{S}_{0x}^{-1} S_{xy}^< = \Sigma_{xz}^{ret} \odot S_{zy}^< + \Sigma_{xz}^< \odot S_{zy}^{adv}$$

Integration over the intermediate spacetime

Green functions  $S^<$  / self-energies  $\Sigma$  :

$$iS_{xy}^< = \eta \langle \{ \Phi^+(y) \Phi(x) \} \rangle$$

$$iS_{xy}^> = \langle \{ \Phi(y) \Phi^+(x) \} \rangle$$

$$iS_{xy}^c = \langle T^c \{ \Phi(x) \Phi^+(y) \} \rangle - \text{causal}$$

$$iS_{xy}^a = \langle T^a \{ \Phi(x) \Phi^+(y) \} \rangle - \text{anticausal}$$

$$S_{xy}^{ret} = S_{xy}^c - S_{xy}^< = S_{xy}^> - S_{xy}^a - \text{retarded}$$

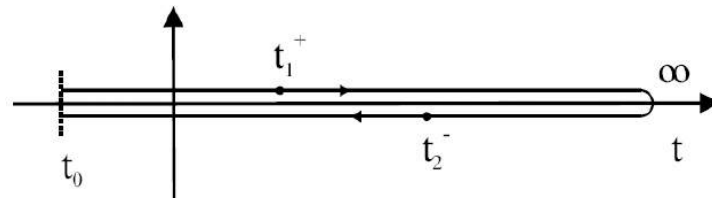
$$S_{xy}^{adv} = S_{xy}^c - S_{xy}^> = S_{xy}^< - S_{xy}^a - \text{advanced}$$

$$\hat{S}_{0x}^{-1} \equiv -(\partial_x^\mu \partial_\mu^x + M_0^2)$$

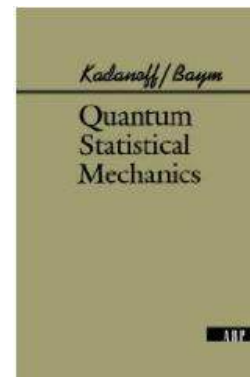
$$\eta = \pm 1 (\text{bosons / fermions})$$

$$T^a (T^c) - (\text{anti-})\text{time - ordering operator}$$

Real-time (Keldysh-) Contour



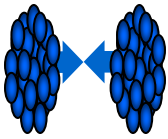
Leo Kadanoff



Gordon Baym

1<sup>st</sup> application for spacially homodeneous system with deformed Fermi sphere:

P. Danielewicz, Ann. Phys. 152, 305 (1984); ... H.S. Köhler, Phys. Rev. 51, 3232 (1995); ...



# From Kadanoff-Baym equations to generalized transport equations

After the first-order gradient expansion of the Wigner transformed Kadanoff-Baym equations and separation into the real and imaginary parts one gets:

## Generalized transport equations (GTE):

$$\text{drift term} \quad \text{Vlasov term} \quad \text{backflow term} \quad \text{collision term} = \text{'gain' - 'loss' term}$$

$$\diamond \{ P^2 - M_0^2 - \text{Re}\Sigma_{XP}^{\text{ret}} \} \{ S_{XP}^< \} - \diamond \{ \Sigma_{XP}^< \} \{ \text{Re}S_{XP}^{\text{ret}} \} = \frac{i}{2} [ \Sigma_{XP}^> S_{XP}^< - \Sigma_{XP}^< S_{XP}^> ]$$

**Backflow term** incorporates the **off-shell** behavior in the particle propagation  
**! vanishes in the quasiparticle limit**  $A_{XP} \rightarrow \delta(p^2 - M^2)$

□ Propagation of the Green's function  $iS_{XP}^< = A_{XP} N_{XP}$ , which carries information not only on the **number of particles** ( $N_{XP}$ ), but also on their **properties**, interactions and correlations (via  $A_{XP}$ )

□ **Spectral function:**

$$A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - \text{Re}\Sigma_{XP}^{\text{ret}})^2 + \Gamma_{XP}^2/4}$$

W. Botermans, R. Malfliet, Phys. Rep. 198 (1990) 115

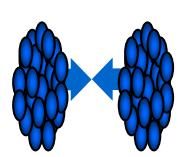
**Reaction rate** of particle (at space-time position X):

$$\Gamma_{XP} = -\text{Im} \Sigma_{XP}^{\text{ret}} = 2 p_0 \Gamma \quad \text{where } \Gamma \text{ is a 'width' of spectral function}$$

4-dimensional generalization of the Poisson-bracket:

$$\diamond \{ F_1 \} \{ F_2 \} := \frac{1}{2} \left( \frac{\partial F_1}{\partial X_\mu} \frac{\partial F_2}{\partial P^\mu} - \frac{\partial F_1}{\partial P_\mu} \frac{\partial F_2}{\partial X^\mu} \right)$$

□ **Life time**  $\tau = \frac{\hbar c}{\Gamma}$



# Generalized testparticle off-shell equations of motion

W. Cassing , S. Juchem, NPA 665 (2000) 377; 672 (2000) 417; 677 (2000) 445

□ Employ **testparticle Ansatz** for the real valued quantity  $i S_{XP}^<$

$$F_{XP} = A_{XP} N_{XP} = i S_{XP}^< \sim \sum_{i=1}^N \delta^{(3)}(\vec{X} - \vec{X}_i(t)) \delta^{(3)}(\vec{P} - \vec{P}_i(t)) \delta(P_0 - \epsilon_i(t))$$

insert in generalized transport equations and determine **equations of motion !**

➔ **Generalized testparticle Cassing-Juchem off-shell equations of motion**  
for the time-like particles:

$$\frac{d\vec{X}_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[ 2\vec{P}_i + \vec{\nabla}_{P_i} \text{Re}\Sigma_{(i)}^{\text{ret}} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\Gamma_{(i)}} \vec{\nabla}_{P_i} \Gamma_{(i)} \right],$$

$$\frac{d\vec{P}_i}{dt} = -\frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[ \vec{\nabla}_{X_i} \text{Re}\Sigma_{(i)}^{\text{ret}} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\Gamma_{(i)}} \vec{\nabla}_{X_i} \Gamma_{(i)} \right],$$

$$\frac{d\epsilon_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[ \frac{\partial \text{Re}\Sigma_{(i)}^{\text{ret}}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\Gamma_{(i)}} \frac{\partial \Gamma_{(i)}}{\partial t} \right],$$

with  $F_{(i)} \equiv F(t, \vec{X}_i(t), \vec{P}_i(t), \epsilon_i(t))$

$$C_{(i)} = \frac{1}{2\epsilon_i} \left[ \frac{\partial}{\partial \epsilon_i} \text{Re}\Sigma_{(i)}^{\text{ret}} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\Gamma_{(i)}} \frac{\partial}{\partial \epsilon_i} \Gamma_{(i)} \right]$$

Realized in PHSD

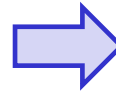


**Note: the common factor  $1/(1-C_{(i)})$  can be absorbed in an ,eigentime‘ of particle (i) !**

# On-shell limits: from KB to BUU

□  $\Gamma(X,P) \rightarrow 0$

$$A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - \text{Re}\Sigma_{XP}^{\text{ret}})^2 + \Gamma_{XP}^2/4}$$



quasiparticle approximation :

$$A_{XP} = 2 p \delta(P^2 - M_0^2)$$

□  $\Gamma(X,P)$  such that

$$\nabla_X \Gamma = 0 \quad \text{and} \quad \nabla_P \Gamma = 0$$



E.g.:  $\Gamma = \text{const}$

$$\Gamma = \Gamma_{\text{vacuum}}(M)$$

,Vacuum' spectral function with constant or mass dependent width  $\Gamma$ :

i.e. spectral function  $A_{XP}$  does **NOT** change the shape (and pole position) during propagation through the medium (no density-, T-dependence)

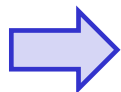


In on-shell limits the **'backflow term'** - which incorporates the off-shell behavior in the particle propagation - **vanishes**:

$$\frac{d\vec{X}_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[ 2\vec{P}_i + \vec{\nabla}_{P_i} \text{Re}\Sigma_{(i)}^{\text{ret}} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\Gamma_{(i)}} \vec{\nabla}_{P_i} \Gamma_{(i)} \right],$$

$$\frac{d\vec{P}_i}{dt} = -\frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[ \vec{\nabla}_{X_i} \text{Re}\Sigma_{(i)}^{\text{ret}} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\Gamma_{(i)}} \vec{\nabla}_{X_i} \Gamma_{(i)} \right],$$

$$\frac{d\epsilon_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[ \frac{\partial \text{Re}\Sigma_{(i)}^{\text{ret}}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - \text{Re}\Sigma_{(i)}^{\text{ret}}}{\Gamma_{(i)}} \frac{\partial \Gamma_{(i)}}{\partial t} \right],$$



Hamilton equations of motion (independent on  $\Gamma$ )  $\rightarrow$  **BUU limit**

# Mean-field potential in off-shell transport models

- **Many-body theory:** Interacting relativistic particles have a **complex self-energy:**

$$\Sigma_{XP}^{ret} = \text{Re } \Sigma_{XP}^{ret} + i \text{Im } \Sigma_{XP}^{ret}$$

The neg. imaginary part  $\Gamma_{XP} = -\text{Im } \Sigma_{XP}^{ret} = 2 p_0 \Gamma$  is related via the width  $\Gamma = \Gamma_{coll} + \Gamma_{dec}$  to the inverse lifetime of the particle  $\tau \sim 1/\Gamma$

- The **collision width**  $\Gamma_{coll}$  is determined from the **loss term** of the collision integral  $I_{coll}$

$$-I_{coll}(loss) = \Gamma_{coll}(X, \vec{P}, M^2) N_{X\vec{P} M^2}$$

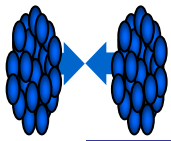
- By **dispersion relation** we get a contribution to the **real part of self-energy:**

$$\text{Re } \Sigma_{XP}^{ret}(p_0) = P \int_0^{\infty} dq \frac{\text{Im } \Sigma_{XP}^{ret}(q)}{(q - p_0)}$$

which gives a **mean-field potential**  $U_{XP}$  via:

$$\text{Re } \Sigma_{XP}^{ret}(p_0) = 2 p_0 U_{XP}$$

→ The **complex self-energy** relates in a self-consistent way to the **self-generated mean-field potential and collision width**



# Collision term in off-shell transport approach

**Collision term for reaction 1+2→3+4:**

spectral functions:

$$I_{coll}(X, \vec{P}, M^2) = Tr_2 Tr_3 Tr_4 \underbrace{A(X, \vec{P}, M^2) A(X, \vec{P}_2, M_2^2) A(X, \vec{P}_3, M_3^2) A(X, \vec{P}_4, M_4^2)}_{\text{spectral functions}}$$

$$\underbrace{|G((\vec{P}, M^2) + (\vec{P}_2, M_2^2) \rightarrow (\vec{P}_3, M_3^2) + (\vec{P}_4, M_4^2))|_{\mathcal{A}, \mathcal{S}}^2}_{\text{transition amplitude}} \delta^{(4)}(P + P_2 - P_3 - P_4)$$

$$[ \underbrace{N_{X\vec{P}_3 M_3^2} N_{X\vec{P}_4 M_4^2} \bar{f}_{X\vec{P} M^2} \bar{f}_{X\vec{P}_2 M_2^2}}_{\text{,gain' term}} - \underbrace{N_{X\vec{P} M^2} N_{X\vec{P}_2 M_2^2} \bar{f}_{X\vec{P}_3 M_3^2} \bar{f}_{X\vec{P}_4 M_4^2}}_{\text{,loss' term}} ]$$

with  $\bar{f}_{X\vec{P} M^2} = 1 + \eta N_{X\vec{P} M^2}$  and  $\eta = \pm 1$  for bosons/fermions, respectively.

The trace over particles 2,3,4 reads explicitly

for fermions

$$Tr_2 = \sum_{\sigma_2, \tau_2} \frac{1}{(2\pi)^4} \int d^3 P_2 \frac{dM_2^2}{2\sqrt{\vec{P}_2^2 + M_2^2}}$$

for bosons

$$Tr_2 = \sum_{\sigma_2, \tau_2} \frac{1}{(2\pi)^4} \int d^3 P_2 \frac{dP_{0,2}^2}{2}$$

additional integration

The off-shell transport approach and the particle spectral functions are fully determined once the **in-medium transition amplitudes G** are known in their **off-shell dependence!**

# Advantages of Kadanoff-Baym dynamics vs Boltzmann

## Kadanoff-Baym equations:

- propagate two-point Green functions  $G^<(x,p) \rightarrow A(x,p) * N(x,p)$  in 8 dimensions  $x=(t,\vec{r})$   $p=(p_0,\vec{p})$
  - $G^<$  carries information not only on the occupation number  $N_{XP}$ , but also on the particle properties, interactions and correlations via spectral function  $A_{XP}$
- Applicable for strong coupling = **strongly interacting system**
  - Dynamically generates a **broad spectral function** for strong coupling
  - Includes **memory effects** (time integration) and **off-shell transitions** in collision term
  - KB can be **solved exactly** for model cases such as  **$\Phi^4$  – theory**
- KB can be **solved in 1<sup>st</sup> order gradient expansion** in terms of generalized transport equations (in test particle ansatz) for **realistic systems of HICs** → PHSD

## Boltzmann equations

- propagate phase space distribution function  $f(\vec{r},\vec{p},t)$  in 6+1 dimensions
- works well for small coupling = weakly interacting system, → **on-shell approach**



# Collision integral: $n \leftrightarrow m$ reactions

# Detailed balance on the level of $2 \leftrightarrow n$ : treatment of multi-particle collisions in transport approaches

W. Cassing, NPA 700 (2002) 618

**Generalized off-shell collision integral for  $n \leftrightarrow m$  reactions:**

$$I_{coll} = \sum_n \sum_m I_{coll}[n \leftrightarrow m]$$

$$\begin{aligned}
 I_{coll}^i[n \leftrightarrow m] = & \\
 & \frac{1}{2} N_n^m \sum_\nu \sum_\lambda \left( \frac{1}{(2\pi)^4} \right)^{n+m-1} \int \left( \prod_{j=2}^n d^4 p_j A_j(x, p_j) \right) \left( \prod_{k=1}^m d^4 p_k A_k(x, p_k) \right) \\
 & \times A_i(x, p) W_{n,m}(p, p_j; i, \nu \mid p_k; \lambda) (2\pi)^4 \delta^4(p^\mu + \sum_{j=2}^n p_j^\mu - \sum_{k=1}^m p_k^\mu) \\
 & \times [\tilde{f}_i(x, p) \prod_{k=1}^m f_k(x, p_k) \prod_{j=2}^n \tilde{f}_j(x, p_j) - f_i(x, p) \prod_{j=2}^n f_j(x, p_j) \prod_{k=1}^m \tilde{f}_k(x, p_k)].
 \end{aligned}$$

$\tilde{f} = 1 + \eta f$  is Pauli-blocking or Bose-enhancement factors;  
 $\eta=1$  for bosons and  $\eta=-1$  for fermions

$W_{n,m}(p, p_j; i, \nu \mid p_k; \lambda)$  is a **transition matrix element squared**

# Multi-meson fusion in heavy-ion reactions

## Multi-meson fusion reactions

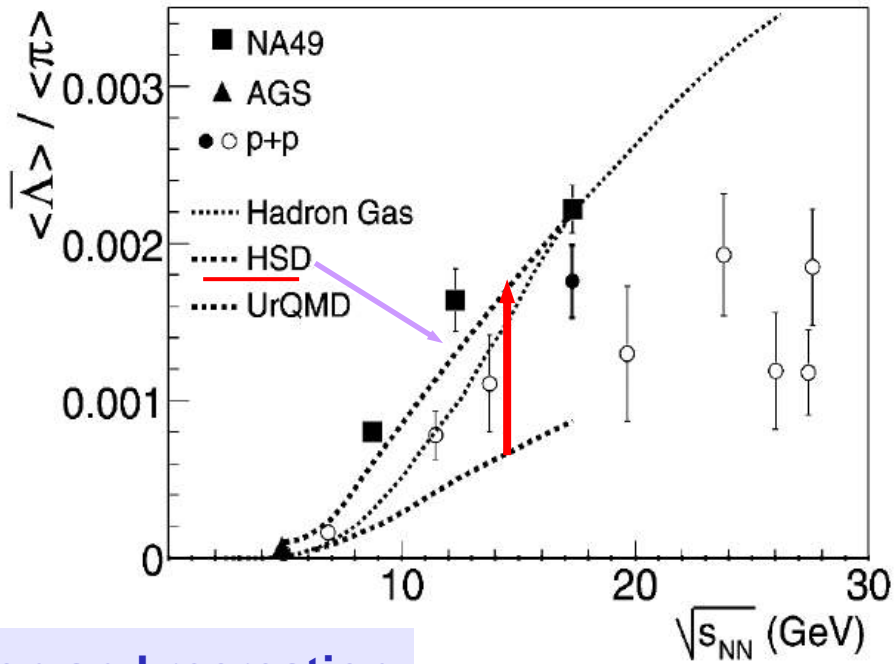
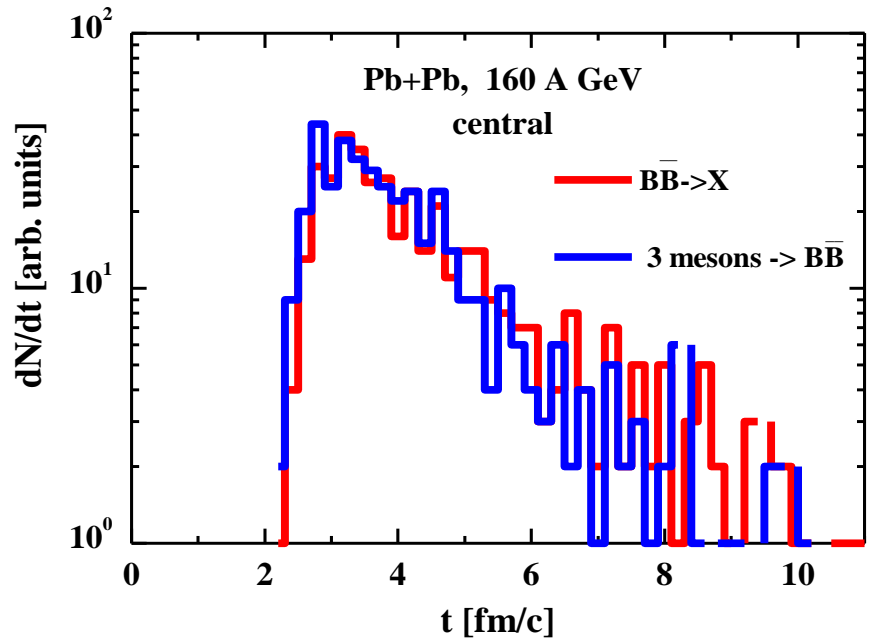
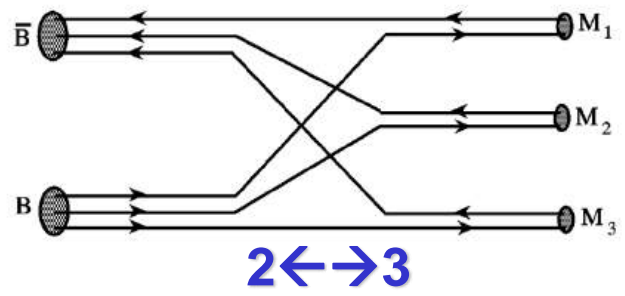
$$m_1 + m_2 + \dots + m_n \leftrightarrow B + B\bar{}$$

$m = \pi, \rho, \omega, \dots$   $B = p, \Lambda, \Sigma, \Xi, \Omega$ , (>2000 channels)

□ important for anti-proton, anti- $\Lambda$ , anti- $\Xi$ , anti- $\Omega$  dynamics !

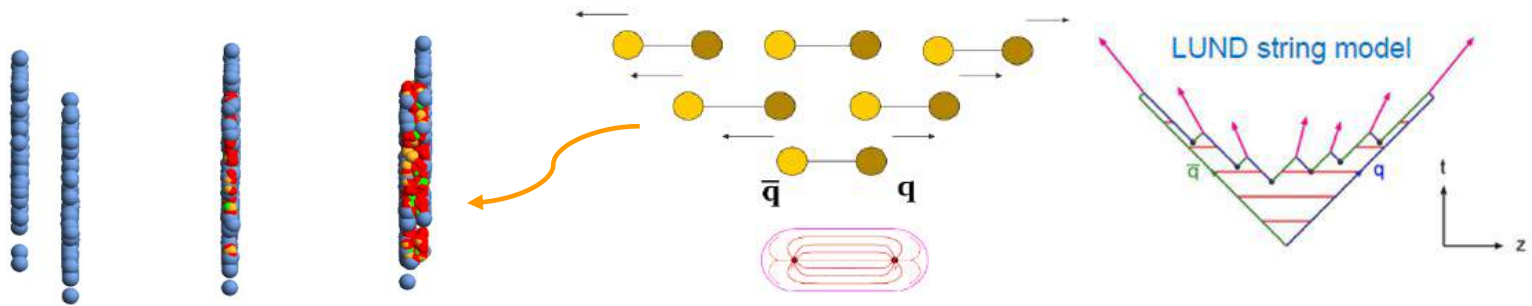
W. Cassing, NPA 700 (2002) 618

E. Seifert, W. Cassing, PRC 97 (2018) 024913, (2018) 044907



→ approximate equilibrium of annihilation and recreation

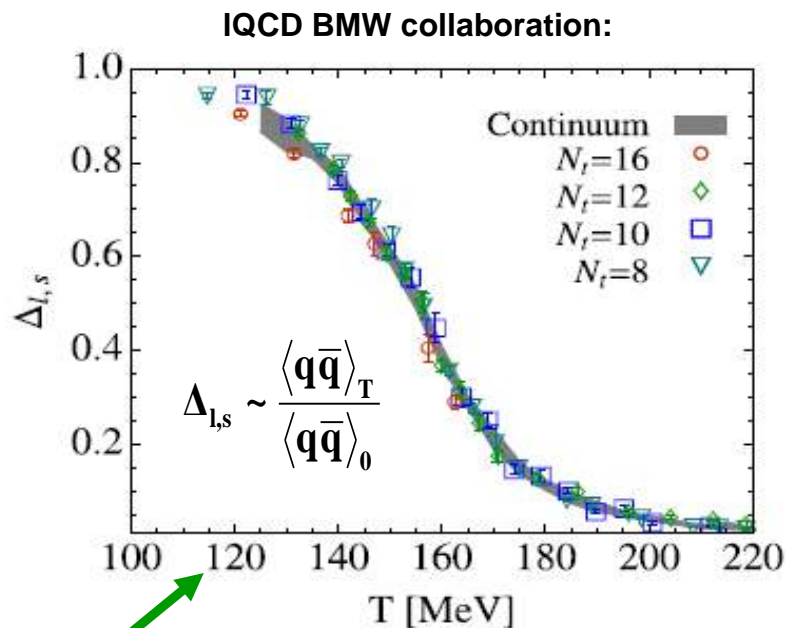
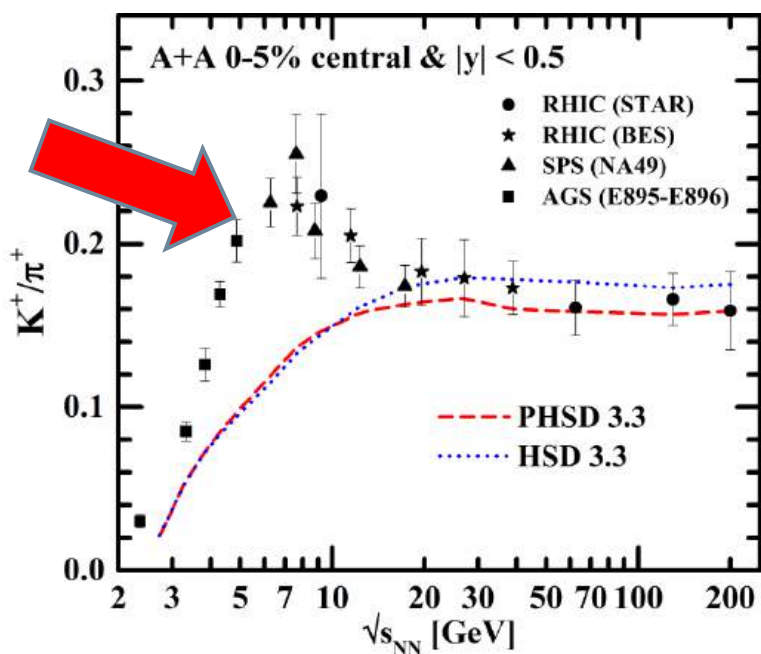
# Modeling of the chiral symmetry restoration via Schwinger mechanism for string fragmentation in the initial phase of HIC



# 'Flavour chemistry' of HIC: $K^+/\pi^+$ ,horn' – 2015

**PHSD:** even when considering the creation of a QGP phase, the  $K^+/\pi^+$  ,horn' seen experimentally by NA49 and STAR at a bombarding energy  $\sim 30$  A GeV (FAIR/NICA energies) remained unexplained (2015)!

→ The origin of the 'horn' is not traced back to deconfinement ?!



Can it be related to **chiral symmetry restoration** in the **initial hadronic phase** ?!



# Scalar quark condensate in HIC

Non-linear  $\sigma - \omega$  model:

$$\frac{\langle q\bar{q} \rangle}{\langle q\bar{q} \rangle_V} = 1 - \frac{\sum_{\pi} \rho_S}{f_{\pi}^2 m_{\pi}^2} - \sum_h \frac{\sigma_h \rho_S^h}{f_{\pi}^2 m_{\pi}^2}$$

baryonic  
medium

mesonic  
medium

PHSD:

Ratio of the scalar quark condensate:

$$\frac{\langle q\bar{q} \rangle}{\langle q\bar{q} \rangle_V}$$

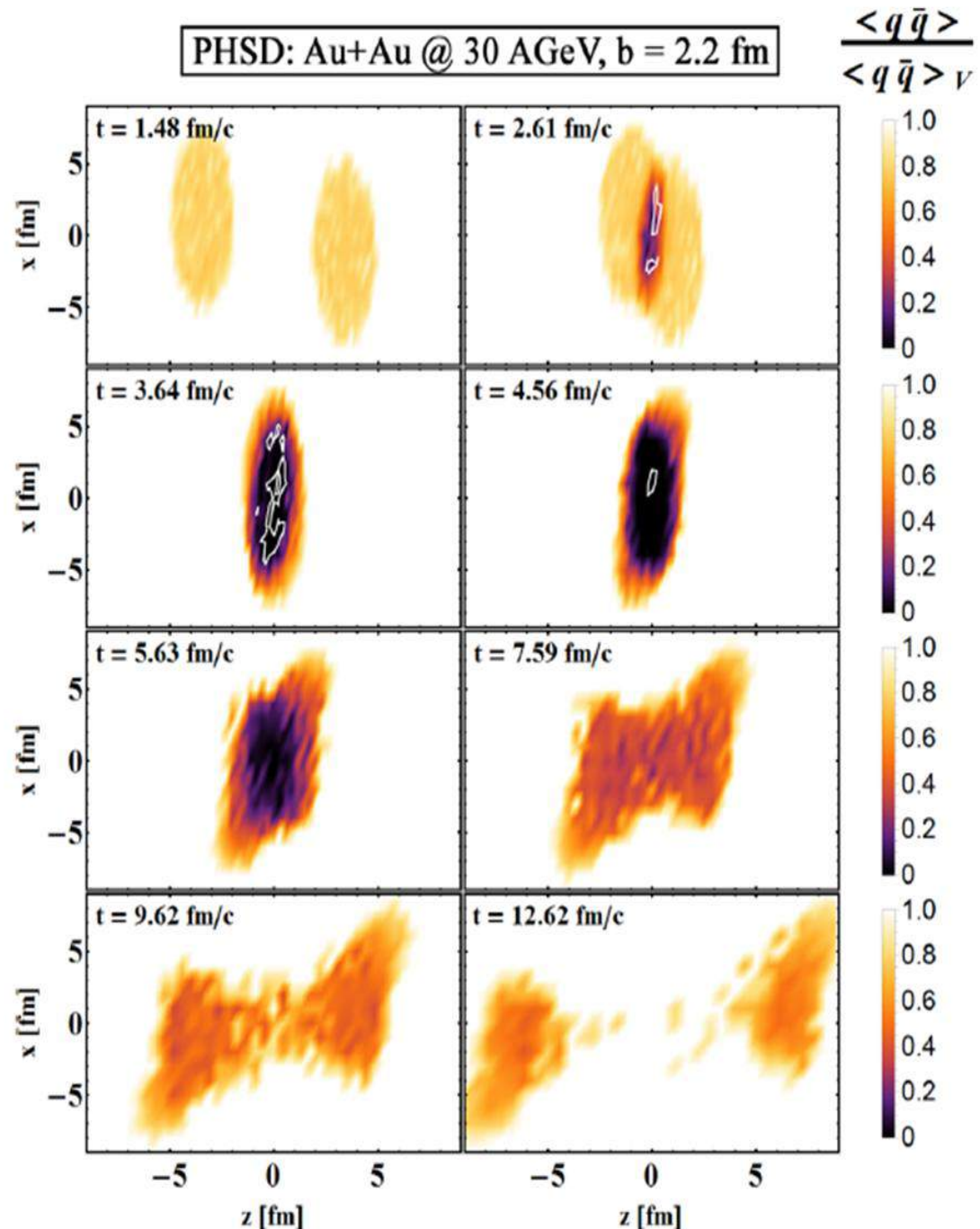


compared to the vacuum value as a function of  $x, z$  ( $y=0$ ) at different time  $t$  for central Au+Au collisions at 30 AGeV

□ restoration of chiral symmetry:

$$\langle q\bar{q} \rangle / \langle q\bar{q} \rangle_V \rightarrow 0$$

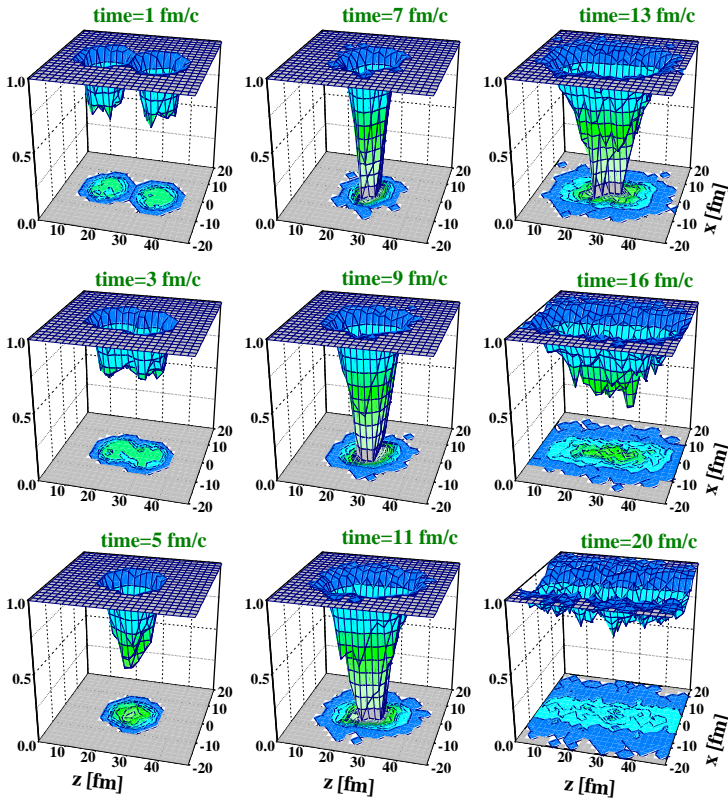
PHSD: Au+Au @ 30 AGeV,  $b = 2.2$  fm



# Quark condensate in central Au+Au

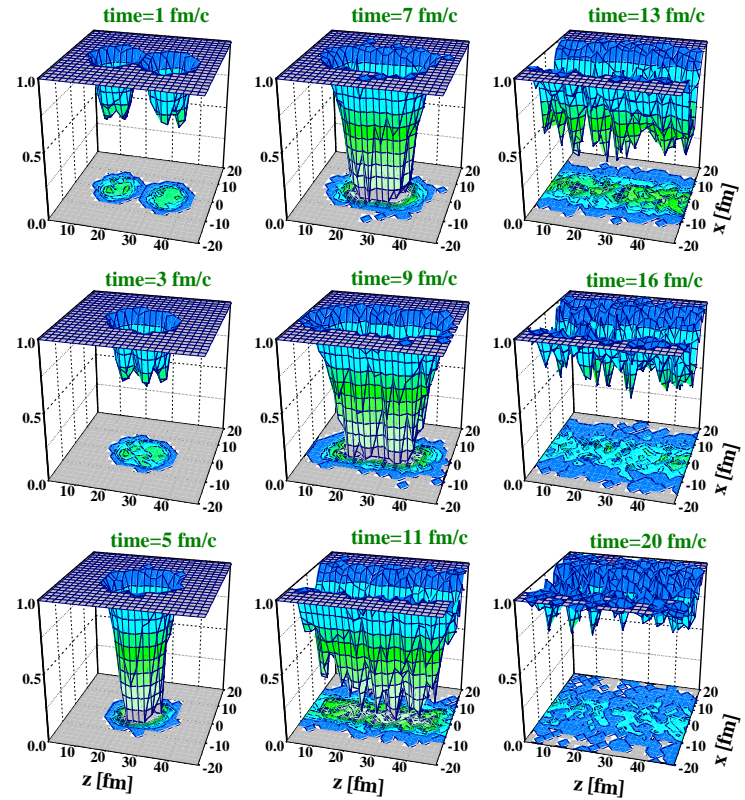
$$\langle \bar{q}q(x,0,z;t) \rangle / \langle \bar{q}q \rangle_v$$

Au+Au, 6 A GeV (central)



$$\langle \bar{q}q(x,0,z;t) \rangle / \langle \bar{q}q \rangle_v$$

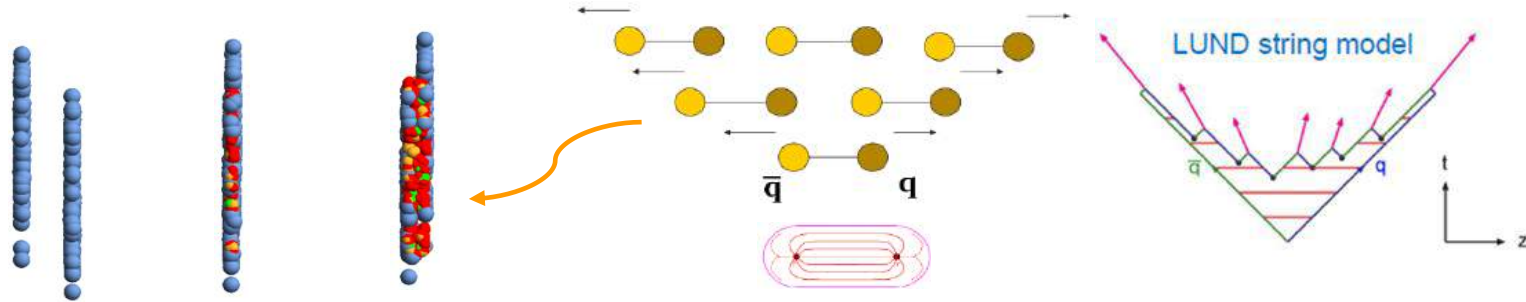
Au+Au, 20 A GeV (central)



- Quark condensate drops to zero already at lower AGS energies!
- → we are probing a **new phase of matter** already at ~5A GeV

# Chiral symmetry restoration via Schwinger mechanism

- Initial stage of HIC: string formation



- the 'flavor chemistry' of the final hadrons in the PHSD is mainly defined by the LUND string model
- 'quark flavor chemistry' in the LUND model is determined by the Schwinger-formula
- According to the Schwinger-formula, the probability to form a massive  $s\bar{s}$  pair in a string-decay is suppressed in comparison to a light flavor pair ( $u\bar{u}$ ,  $d\bar{d}$ ):

$$\frac{P(s\bar{s})}{P(u\bar{u})} = \frac{P(s\bar{s})}{P(d\bar{d})} = \gamma_s = \exp\left(-\pi \frac{m_s^2 - m_q^2}{2\kappa}\right)$$

with  $\kappa$ - string tension;  
in vacuum:  $\kappa \sim 0.9 \text{ GeV/fm} = 0.176 \text{ GeV}^2$

- $m_s, m_q$  ( $q=u,d$ ) – constituent ('dressed') quark masses

# Dressing of the quark masses

- $m_s, m_q (q=u,d)$  – **constituent** ('dressed') quark masses: 'dressing' of bare quark masses is due to the coupling to the **scalar quark condensate**  $\langle q\bar{q} \rangle$  :

## I. In vacuum (e.g. p+p collisions) :

$$m_q^V = m_q^0 - g_s \langle q\bar{q} \rangle_V$$

( $V \equiv \text{vacuum}$ )

**bare quark masses:**

$$m_u^0 = m_d^0 \approx 7 \text{ MeV}, \quad m_s^0 \approx 100 \text{ MeV}$$

**vacuum scalar quark condensate**

**fixed from Gell-Mann-Oakes-Renner**

**relation**  $f_\pi^2 m_\pi^2 = -\frac{1}{2}(m_u^0 + m_d^0) \langle \bar{q}q \rangle_V$

$\Rightarrow \langle q\bar{q} \rangle_V \approx -3.2 \text{ fm}^{-3}$

**$\rightarrow$  Constituent quark masses in vacuum :**

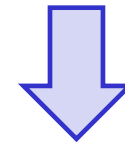
( $m_q \equiv m_q^V$ )  $m_u^V = m_d^V \approx 0.35 \text{ GeV}, \quad m_s^V \approx 0.5 \text{ GeV}$

## II. In medium (e.g. A+A collisions) :

In the presence of a **hot and dense hadronic medium**, the degrees of freedom modify their properties, e.g. **the in-medium constituent quark masses:**

$$m_q^* = m_q^0 - g_s \langle q\bar{q} \rangle$$

( $q = u, d, s$ )



$$m_q^* = m_q^0 + (m_q^V - m_q^0) \frac{\langle q\bar{q} \rangle}{\langle q\bar{q} \rangle_V}$$

\* mean-field results (1PI)



# Scalar quark condensate in the hadronic medium

- The behavior of the scalar quark condensate  $\langle q\bar{q} \rangle$  in the **hadronic medium** (baryons + mesons) can be obtained e.g. from

B. Friman et al., Eur. Phys. J. A 3, 165, 1998

**non-linear  $\sigma - \omega$  model:**

$$\frac{\langle q\bar{q} \rangle}{\langle q\bar{q} \rangle_V} = 1 - \frac{\Sigma_\pi}{f_\pi^2 m_\pi^2} \rho_S - \sum_h \frac{\sigma_h \rho_S^h}{f_\pi^2 m_\pi^2}$$

**baryonic medium**

**mesonic medium**

where  $\Sigma_\pi \approx 45 \text{ MeV}$

is the pion-nucleon  $\Sigma$ -term,

$\sigma_h = m_\pi/2$  for light mesons;  
 $= m_\pi/4$  - strange mesons

Scalar field  $\sigma(x)$  mediates the scalar interaction of baryons with the surrounding medium with a  $g_s$  coupling

- 1)  $\rho_s$  is the **scalar density of baryonic matter** :

from non-linear  $\sigma - \omega$  model:

from PHSD

$$m_\sigma^2 \sigma(x) + B\sigma^2(x) + C\sigma^3(x) = g_s \rho_S = g_s d \int \frac{d^3 p}{(2\pi)^3} \frac{m_N^*(x)}{\sqrt{p^2 + m_N^{*2}}} f_N(x, \mathbf{p})$$

$$m_N^*(x) = m_N^V - g_s \sigma(x)$$

- $\sigma(x)$  is determined locally by solution of the **nonlinear gap equation** ;
- parameters  $g_s, m_\sigma, B, C$  are **fixed** to reproduce the main nuclear matter quantities, i.e. saturation density, binding energy per nucleon, compression modulus and the effective nucleon mass.

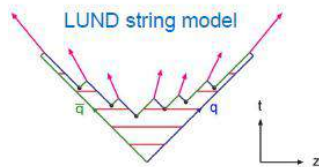
- 2)  $\rho_s^h$  is the **scalar density of mesons** of type  $h \rightarrow$  from PHSD

# Chiral symmetry restoration vs. deconfinement



## I. Initial stage of HICs:

Hadronic matter  $\rightarrow$  string formation



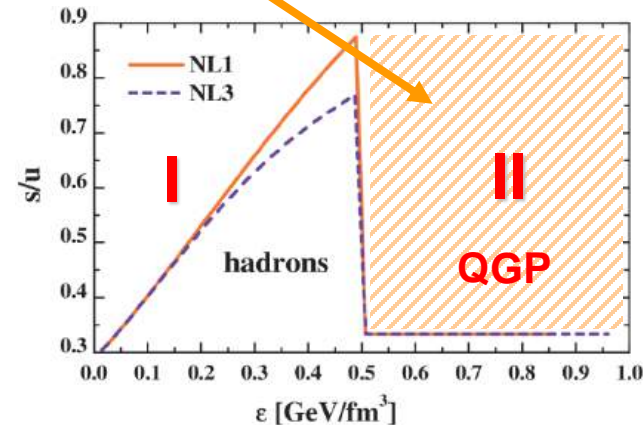
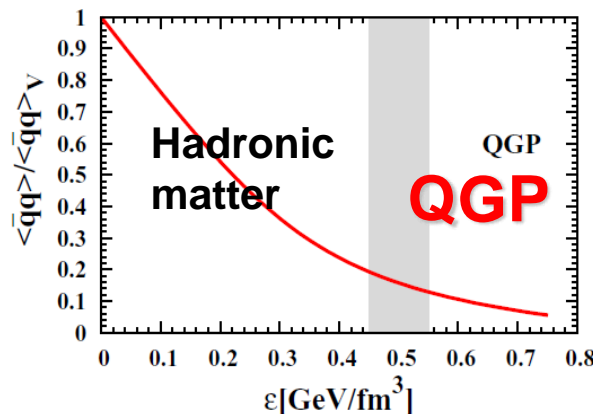
$$\frac{P(s\bar{s})}{P(u\bar{u})} = \frac{P(s\bar{s})}{P(d\bar{d})} = \gamma_s = \exp\left(-\pi \frac{m_s^{*2} - m_q^{*2}}{2\kappa}\right)$$

$$m_q^* = m_q^0 + (m_q^v - m_q^0) \frac{\langle q\bar{q} \rangle}{\langle q\bar{q} \rangle_v}$$

## II. QGP

(time-like partons, explicit partonic interactions)

## III. Hadronic phase



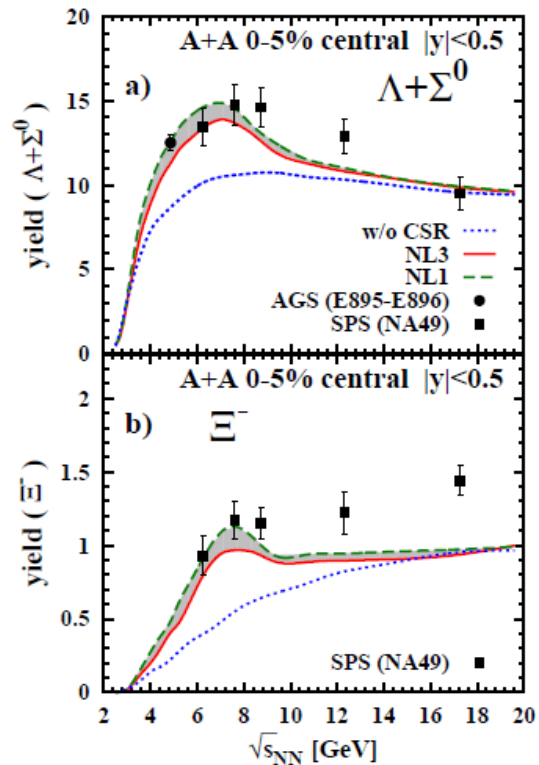
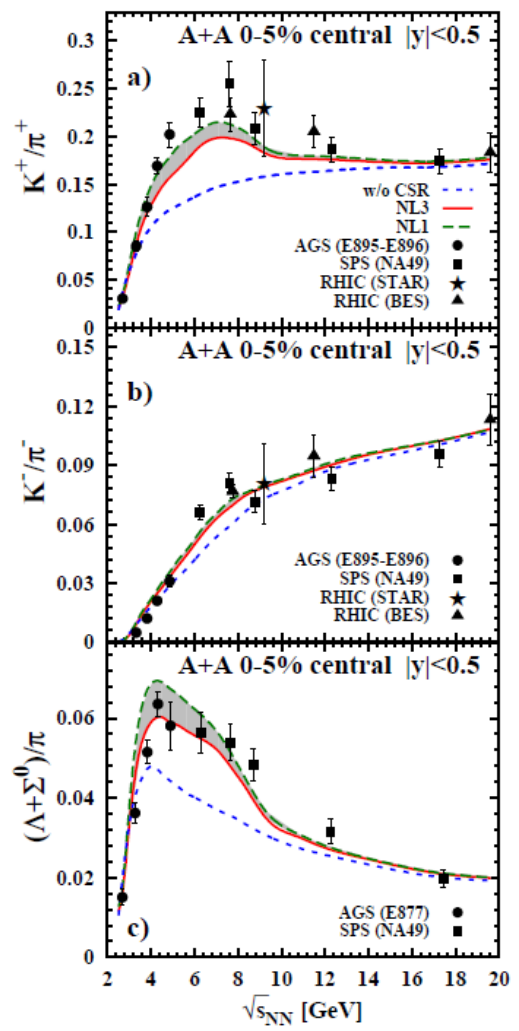
□ Chiral symmetry restoration via Schwinger mechanism (and non-linear  $\sigma - \omega$  model) changes the „flavour chemistry“ in string fragmentation (1PI):

$$\langle q\bar{q} \rangle / \langle q\bar{q} \rangle_v \rightarrow 0 \quad \rightarrow \quad m_s^* \rightarrow m_s^0 \quad \rightarrow \quad s/u \text{ grows}$$

$\rightarrow$  the strangeness production probability increases with the local energy density  $\epsilon$  (up to  $\epsilon_c$ ) due to the partial chiral symmetry restoration!

# Excitation function of hadron ratios and yields

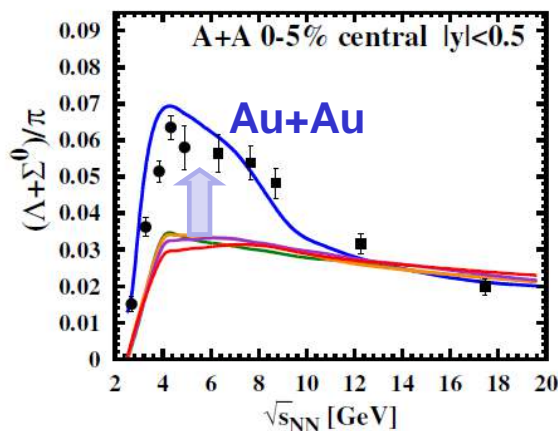
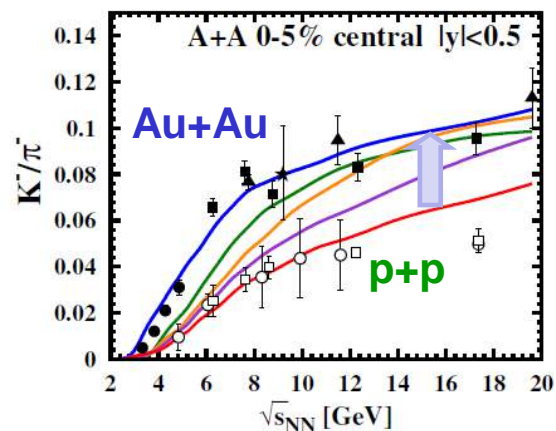
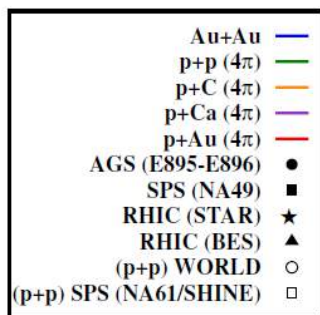
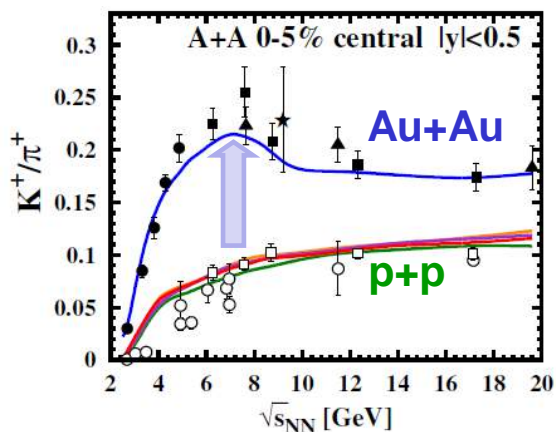
A. Palmese et al., PRC94 (2016) 044912, arXiv:1607.04073



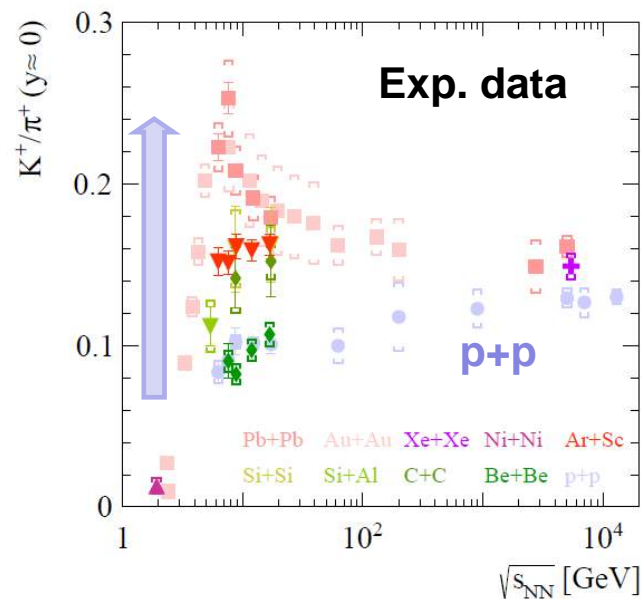
- Influence of EoS: NL1 vs NL3 → **low sensitivity to the nuclear EoS**
- Excitation function of the **hyperons**  $\Lambda+\Sigma^0$  and  $\Xi^-$  show analogous peaks as  $K^+/\pi^+$ ,  $(\Lambda+\Sigma^0)/\pi$  ratios due to CSR

**Chiral symmetry restoration** leads to the **enhancement of strangeness production** in string fragmentation in the beginning of HICs in the hadronic phase.  
**→ The „horn“ structure is due to the interplay between CSR and deconfinement (QGP)**

# Sensitivity to the system size: p+p, p+A, A+A



Excitation function of hadron yields is **very different** in p+p and A+A

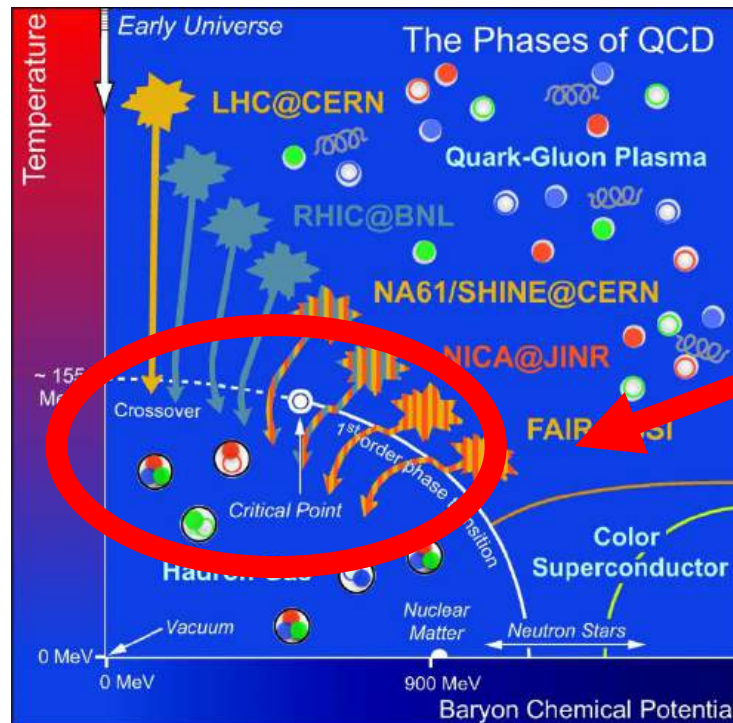
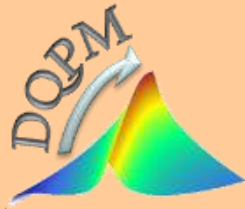


p+p, p+A: **monotonic** increase with energy

A+A: „**horn**“ structure is due to the interplay between **chiral symmetry restoration** and **deconfinement (QGP)** - within the PHSD

# Modeling of sQGP →

## DQPM ( $T, \mu_q$ )



**finite  $T, \mu_q$**

# pQCD: shear viscosity $\eta$

## QCD: Pure Yang-Mills (only gluons)

LO (Leading order) perturbative QCD calculations:

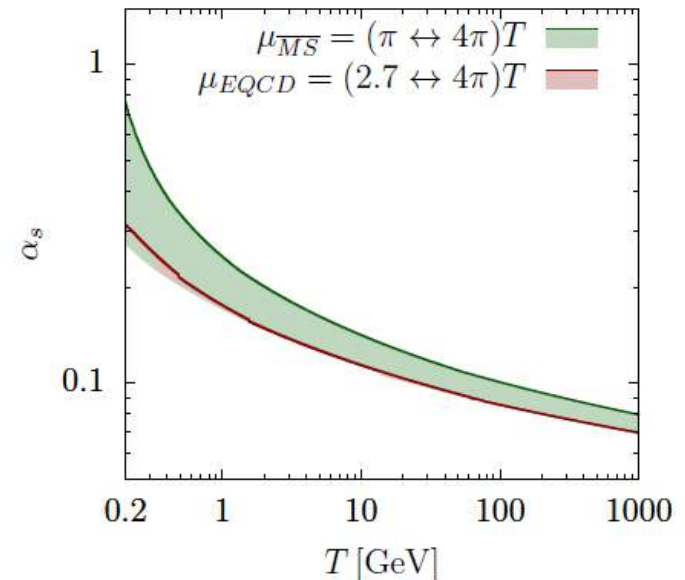
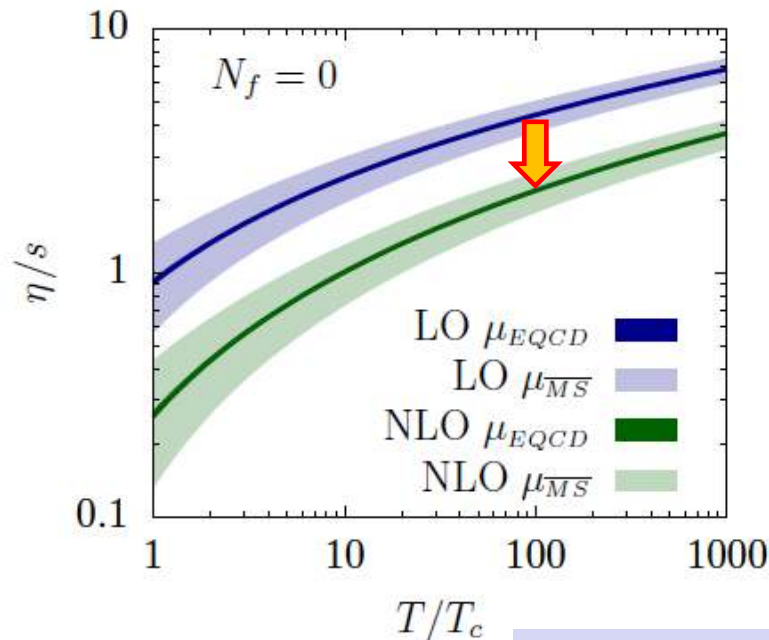
$\eta/s > 0.5$  at  $T$  near  $T_c$  'AMY': P.B. Arnold, G.D. Moore and L.G. Yaffe, JHEP 11 (2000) 001)

NLO (Next-to-leading order):

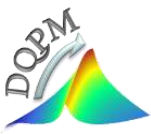
J. Ghiglieri, G.D. Moore, D. Teaney, JHEP 1803 (2018) 179 :

“The next-to-leading order corrections are large and bring  $\eta/s$  down by more than a factor of 3 at physically relevant couplings.

The perturbative expansion is problematic even at  $T \sim 100$  GeV”



→ from pQCD to effective models of QCD!



# Dynamical QuasiParticle Model (DQPM)

**DQPM** – effective model for the description of **non-perturbative** (strongly interacting) QCD based on **IQCD EoS**

**Degrees-of-freedom:** strongly interacting **dynamical quasiparticles** - quarks and gluons

**Theoretical basis :**

□ ,resummed‘ single-particle Green‘s functions → quark (gluon) propagator (2PI) :

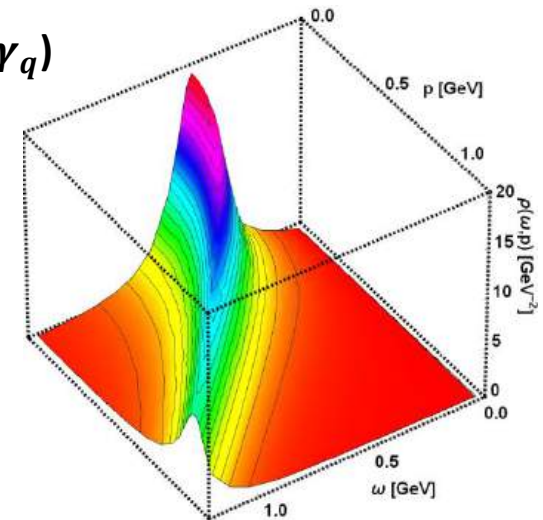
$$\begin{aligned} \text{gluon propagator: } \Delta^{-1} = P^2 - \Pi \quad & \& \quad \text{quark propagator } S_q^{-1} = P^2 - \Sigma_q \\ \text{gluon self-energy: } \Pi = M_g^2 - i2\gamma_g\omega \quad & \& \quad \text{quark self-energy: } \Sigma_q = M_q^2 - i2\gamma_q\omega \end{aligned}$$

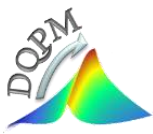
**Properties of the quasiparticles** are specified by scalar **complex self-energies:**

$Re\Sigma_q$  : **thermal masses** ( $M_g, M_q$ );  $Im\Sigma_q$  : **interaction widths** ( $\gamma_g, \gamma_q$ )

→ spectral functions  $\rho_q = -2ImS_q$  → Lorentzian form:

$$\begin{aligned} \rho_j(\omega, \mathbf{p}) &= \frac{\gamma_j}{\tilde{E}_j} \left( \frac{1}{(\omega - \tilde{E}_j)^2 + \gamma_j^2} - \frac{1}{(\omega + \tilde{E}_j)^2 + \gamma_j^2} \right) \\ &\equiv \frac{4\omega\gamma_j}{(\omega^2 - \mathbf{p}^2 - M_j^2)^2 + 4\gamma_j^2\omega^2} \quad \tilde{E}_j^2(\mathbf{p}) = \mathbf{p}^2 + M_j^2 - \gamma_j^2 \end{aligned}$$





# Parton properties

- Modeling of the quark/gluon **masses** and **widths** (ansatz inspired by HTL calculations)

## Masses:

$$M_{q(\bar{q})}^2(T, \mu_B) = \frac{N_c^2 - 1}{8N_c} g^2(T, \mu_B) \left( T^2 + \frac{\mu_q^2}{\pi^2} \right)$$

$$M_g^2(T, \mu_B) = \frac{g^2(T, \mu_B)}{6} \left( \left( N_c + \frac{1}{2} N_f \right) T^2 + \frac{N_c}{2} \sum_q \frac{\mu_q^2}{\pi^2} \right)$$

## Widths:

$$\gamma_{q(\bar{q})}(T, \mu_B) = \frac{1}{3} \frac{N_c^2 - 1}{2N_c} \frac{g^2(T, \mu_B) T}{8\pi} \ln \left( \frac{2c}{g^2(T, \mu_B)} + 1 \right)$$

$$\gamma_g(T, \mu_B) = \frac{1}{3} N_c \frac{g^2(T, \mu_B) T}{8\pi} \ln \left( \frac{2c}{g^2(T, \mu_B)} + 1 \right)$$

- **Coupling g:**  $\frac{\partial}{\partial T} \left( \frac{S_{DQPM}}{T^3} \right) = 0$

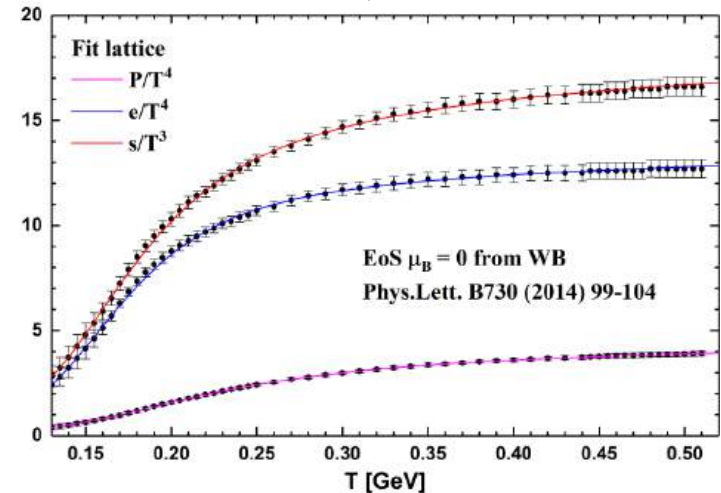
**IQCD entropy density s function of T at  $\mu_B=0$**

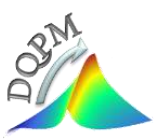
$$g^2(s/s_{SB}) = d \left( (s/s_{SB})^e - 1 \right)^f$$

$$s_{SB}^{QCD} = 19/9 \pi^2 T^3$$

→ **DQPM :**

only **one parameter** ( $c = 14.4$ )  
+  $(T, \mu_B)$ - dependent **coupling constant** has to be determined from lattice results





# DQPM thermodynamics at finite $(T, \mu_q)$

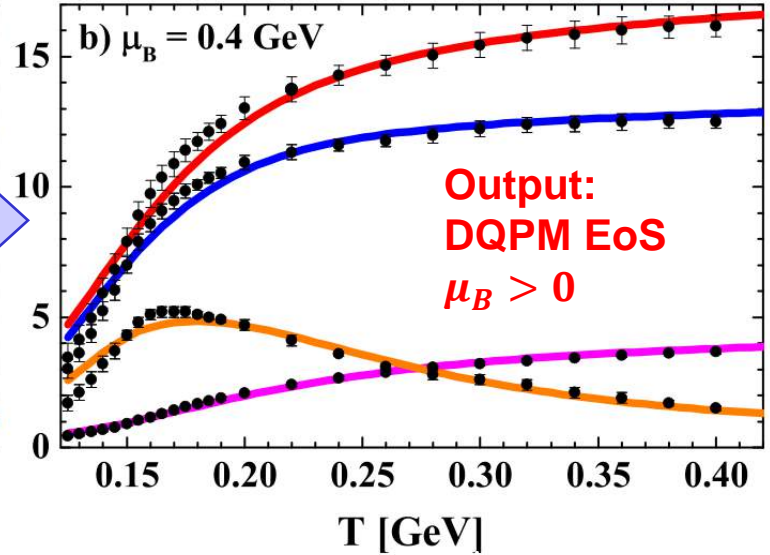
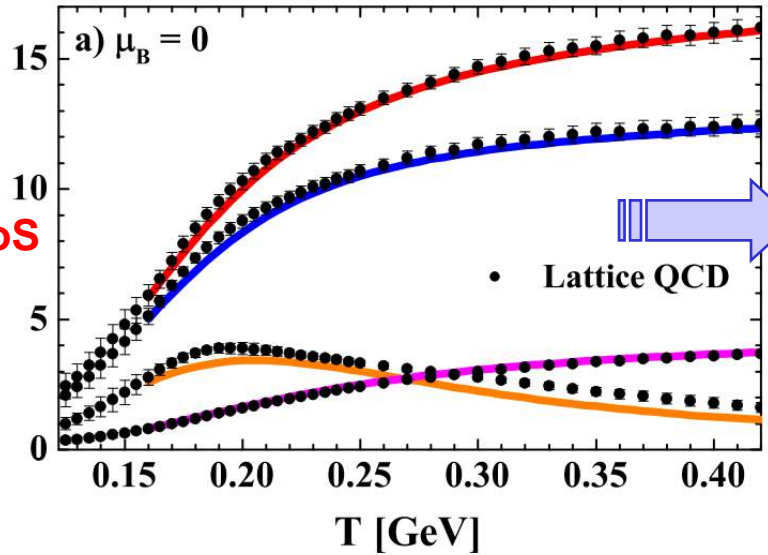
- Entropy and baryon density in the quasiparticle limit (G. Baym 1998):

$$s^{dqp} = - \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \left[ d_g \frac{\partial n_B}{\partial T} (\text{Im}(\ln -\Delta^{-1}) + \text{Im} \Pi \text{Re} \Delta) \right. \\ \left. + \sum_{q=u,d,s} d_q \frac{\partial n_F(\omega - \mu_q)}{\partial T} (\text{Im}(\ln -S_q^{-1}) + \text{Im} \Sigma_q \text{Re} S_q) \right. \\ \left. + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} d_{\bar{q}} \frac{\partial n_F(\omega + \mu_q)}{\partial T} (\text{Im}(\ln -S_{\bar{q}}^{-1}) + \text{Im} \Sigma_{\bar{q}} \text{Re} S_{\bar{q}}) \right]$$

$$n^{dqp} = - \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \left[ \sum_{q=u,d,s} d_q \frac{\partial n_F(\omega - \mu_q)}{\partial \mu_q} (\text{Im}(\ln -S_q^{-1}) + \text{Im} \Sigma_q \text{Re} S_q) \right. \\ \left. + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} d_{\bar{q}} \frac{\partial n_F(\omega + \mu_q)}{\partial \mu_q} (\text{Im}(\ln -S_{\bar{q}}^{-1}) + \text{Im} \Sigma_{\bar{q}} \text{Re} S_{\bar{q}}) \right]$$

B. Vanderheyden, G. Baym, J. Stat. Phys. 93 (1998) 843  
Blazot, Iancu, Rebhan, Phys. Rev. D 63 (2001) 065003

**DQPM:** —  $P/T^4$  —  $\varepsilon/T^4$  —  $s/T^3$  —  $I/T^4$

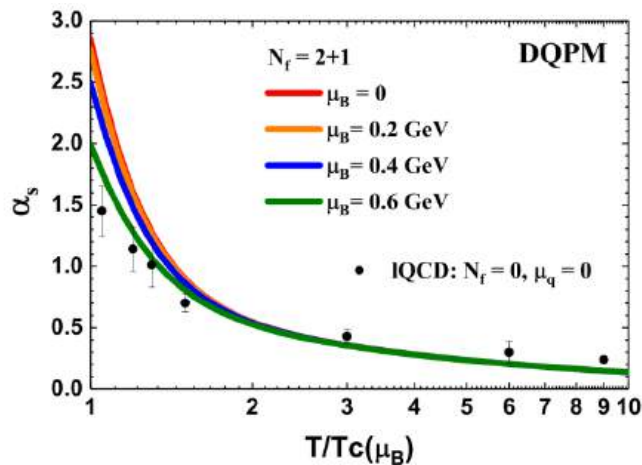


Input:  
lattice EoS  
 $\mu_B = 0$

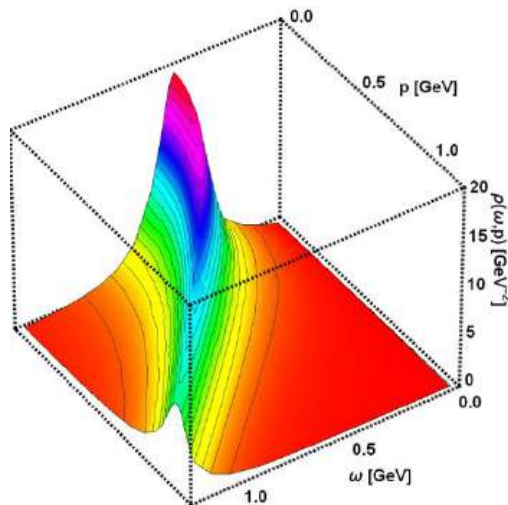
Output:  
DQPM EoS  
 $\mu_B > 0$

# DQPM: parton properties

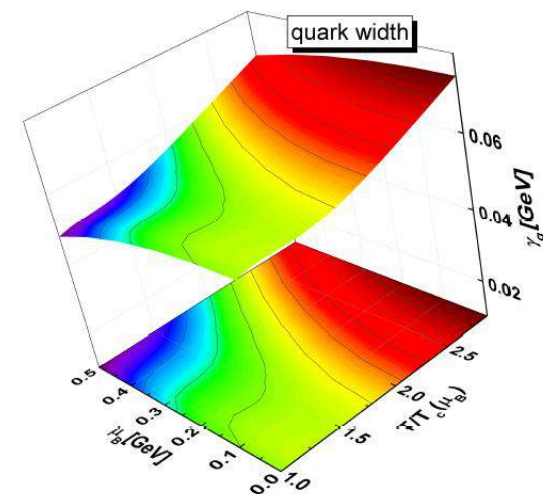
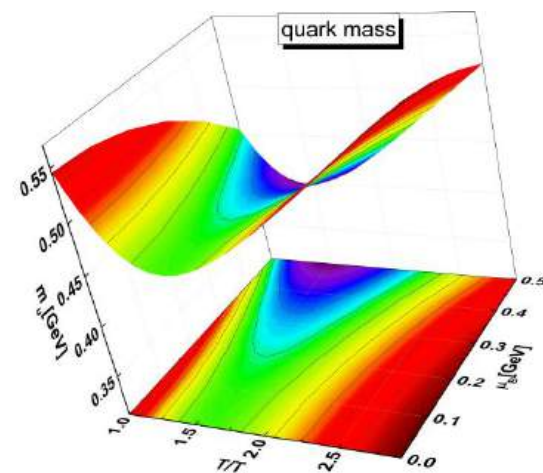
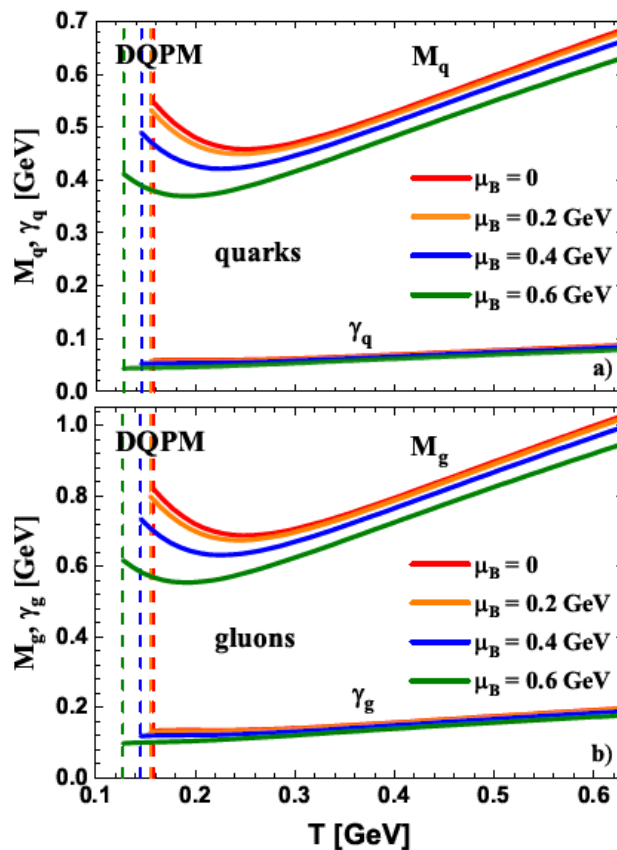
Coupling as a function of  $(T, \mu_B)$



→ Lorentzian spectral function:



Pole masses and widths vs  $(T, \mu_B)$



P. Moreau et al., PRC100 (2019) 014911;  
 O. Soloveva et al., PRC110 (2020) 045203

# Partonic interactions: matrix elements

DQPM partonic cross sections  $\rightarrow$  **leading order diagrams**

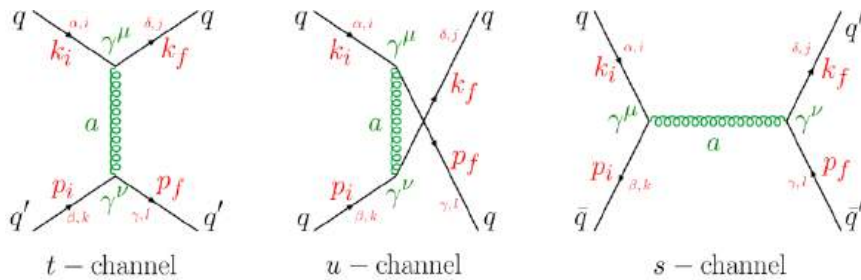
□ **Propagators** for massive bosons and fermions:

$$\frac{\overset{\mu, a}{\text{-----}} \overset{\nu, b}{\text{-----}}}{q} = -i\delta_{ab} \frac{g^{\mu\nu} - q^\mu q^\nu / M_g^2}{q^2 - M_g^2 + 2i\gamma_g q_0}$$

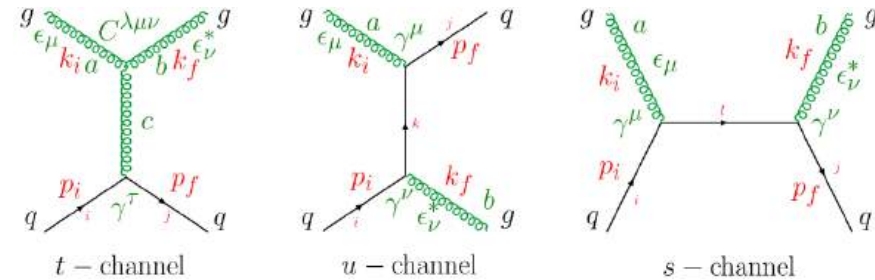
□ **(Quasi-) elastic channels:**

**qq'  $\rightarrow$  qq' scattering**

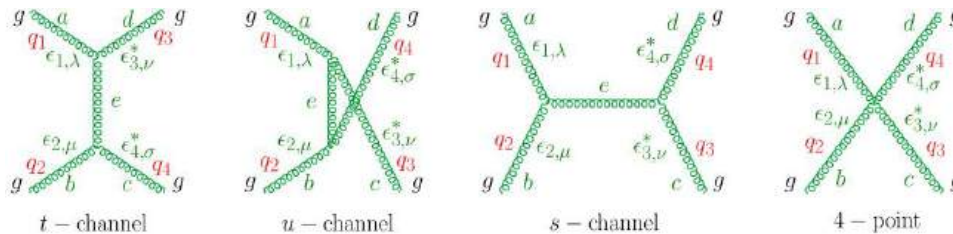
$$\overset{i}{\text{-----}} \overset{j}{\text{-----}}}{q} = i\delta_{ij} \frac{\not{q} + M_q}{q^2 - M_q^2 + 2i\gamma_q q_0}$$



**gq  $\rightarrow$  gq scattering**



**gg  $\rightarrow$  gg scattering**



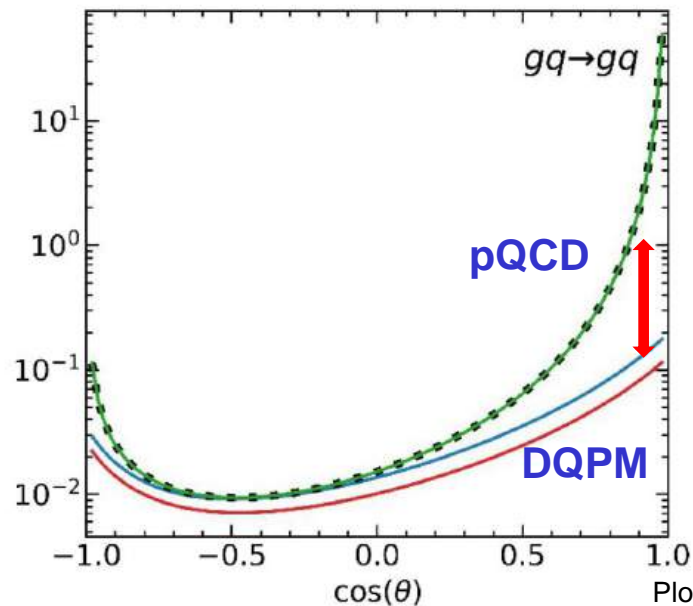
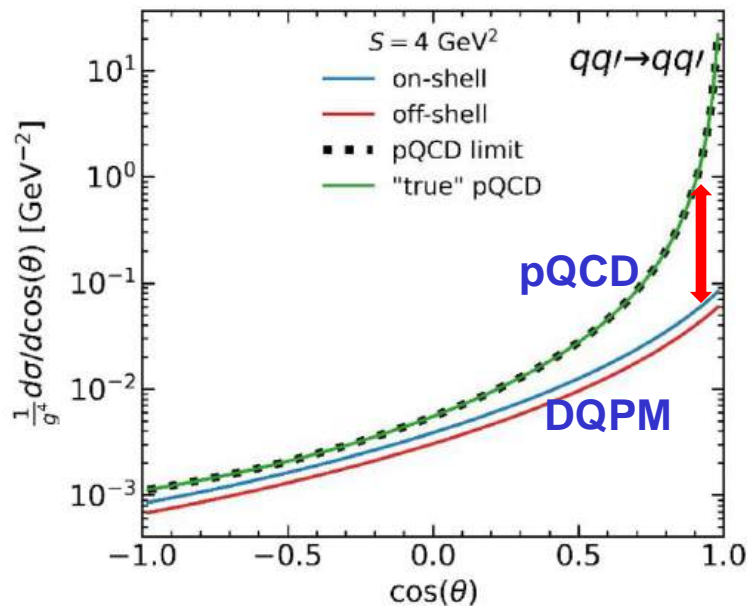
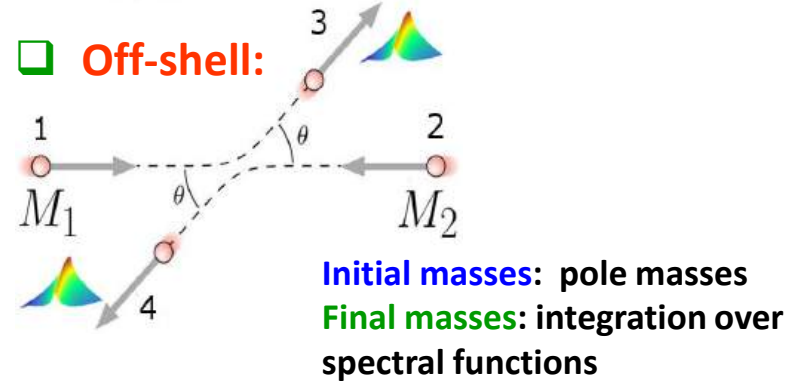
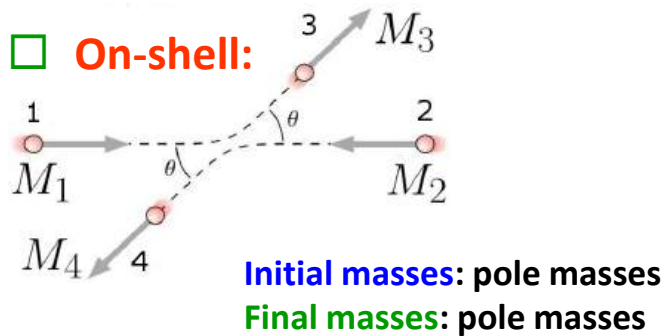
□ **Inelastic channels:**

$$q + \bar{q} \rightarrow g$$

$$g \rightarrow q + \bar{q}$$



# Differential cross sections

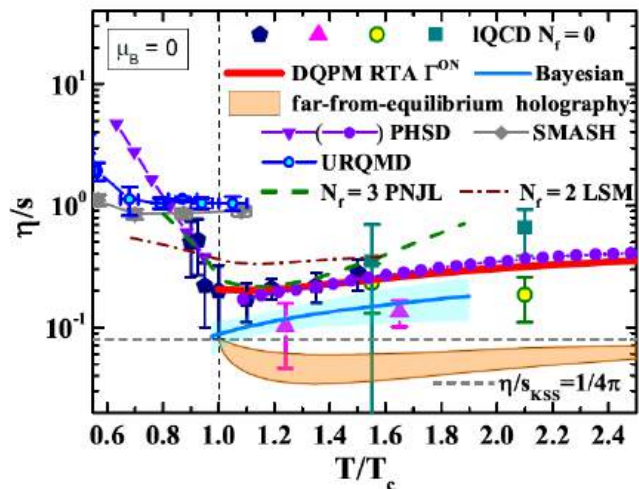


Plot: Ilya Grishmanovskii

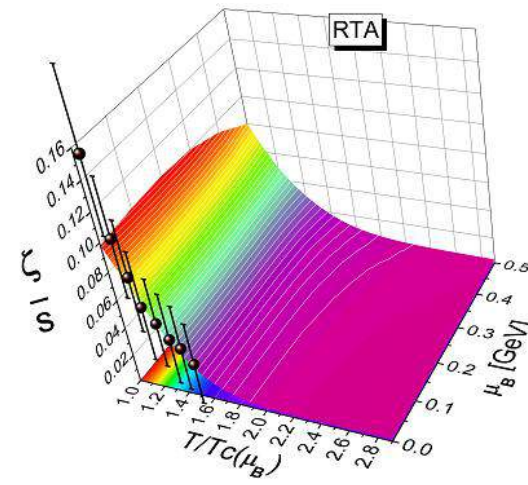
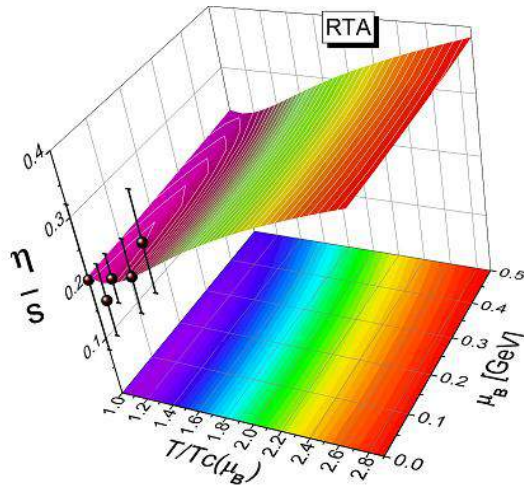
- DQPM:  $M \rightarrow 0, \gamma \rightarrow 0 \rightarrow$  reproduces pQCD limits
- Differences between DQPM and pQCD : less forward peaked angular distribution leads to more efficient momentum transfer

# Transport coefficients

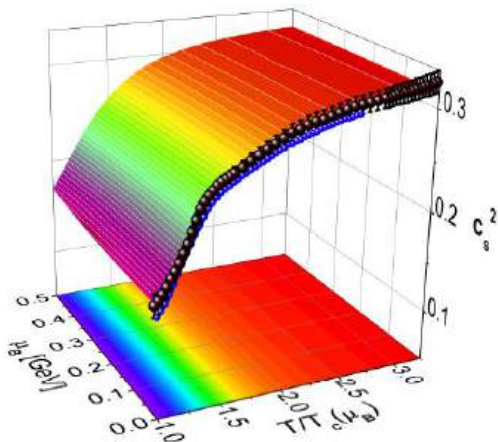
## $\eta/s$ versus $(T, \mu_B)$



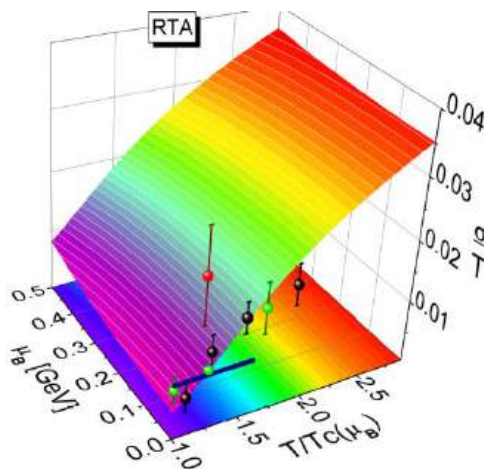
## Bulk viscosity $\zeta/s$



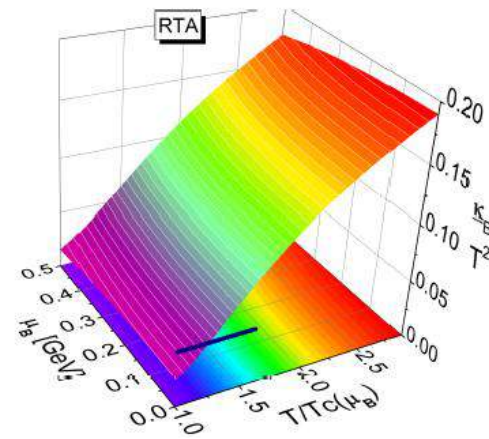
## Speed of sound $c_s^2$



## Electric conductivity $\sigma_e/T$

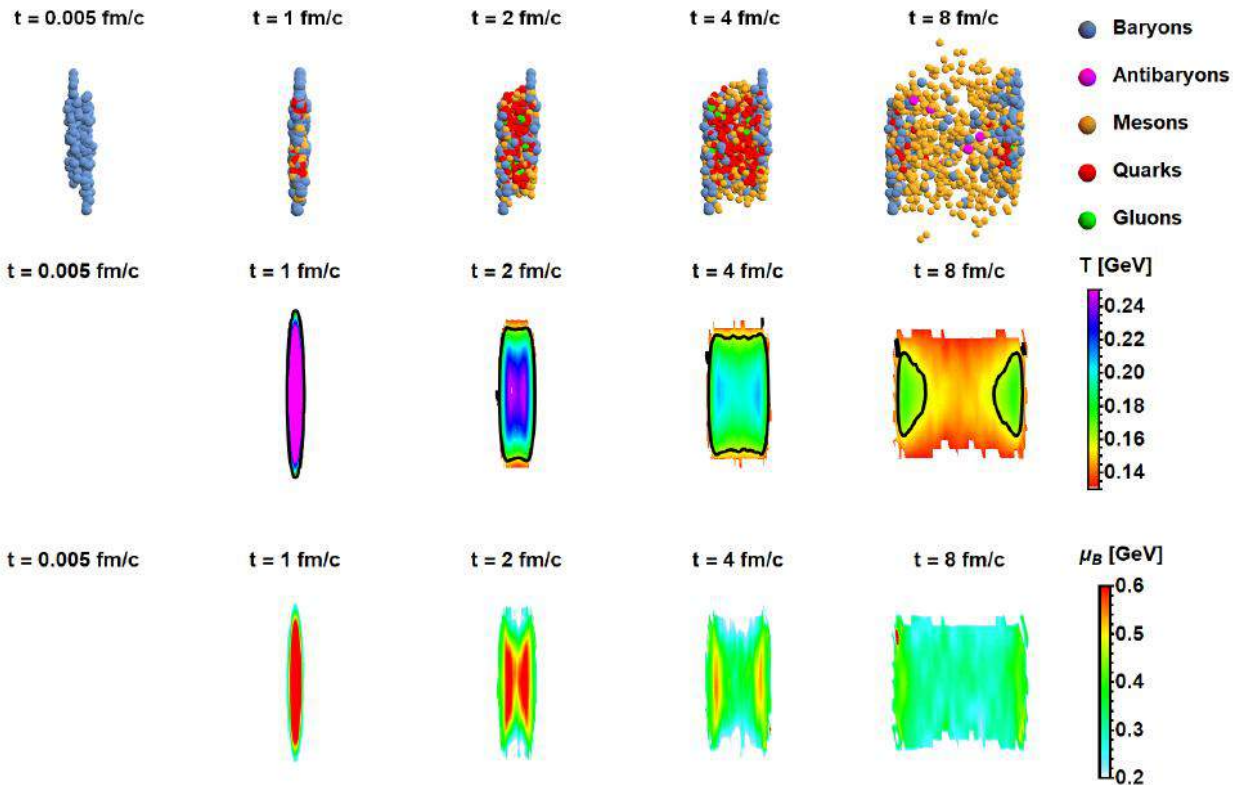


## Baryon diffusion coefficient $\kappa_B/T^2$



# Traces of the QGP at finite $\mu_q$ in observables in high energy heavy-ion collisions

Au + Au  $\sqrt{s_{NN}} = 19.6$  GeV –  $b = 2$  fm – Section view





# PHSD: QGP evolution in HICs

**Input:**  
 $\epsilon^{\text{PHSD}}$  and  $n_B^{\text{PHSD}}$

**IQCD**

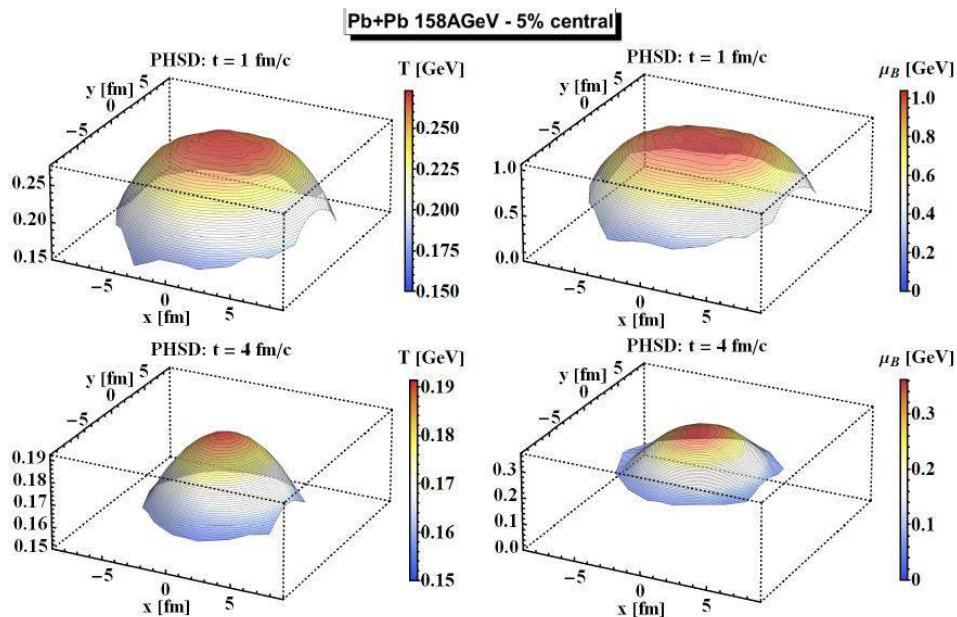
$$\frac{n_B}{T^3} \approx \chi_2^B(T) \left(\frac{\mu_B}{T}\right) + \dots$$

$$\Delta\epsilon/T^4 \approx \frac{1}{2} \left( T \frac{\partial \chi_2^B(T)}{\partial T} + 3\chi_2^B(T) \right) \left(\frac{\mu_B}{T}\right)^2 + \dots$$

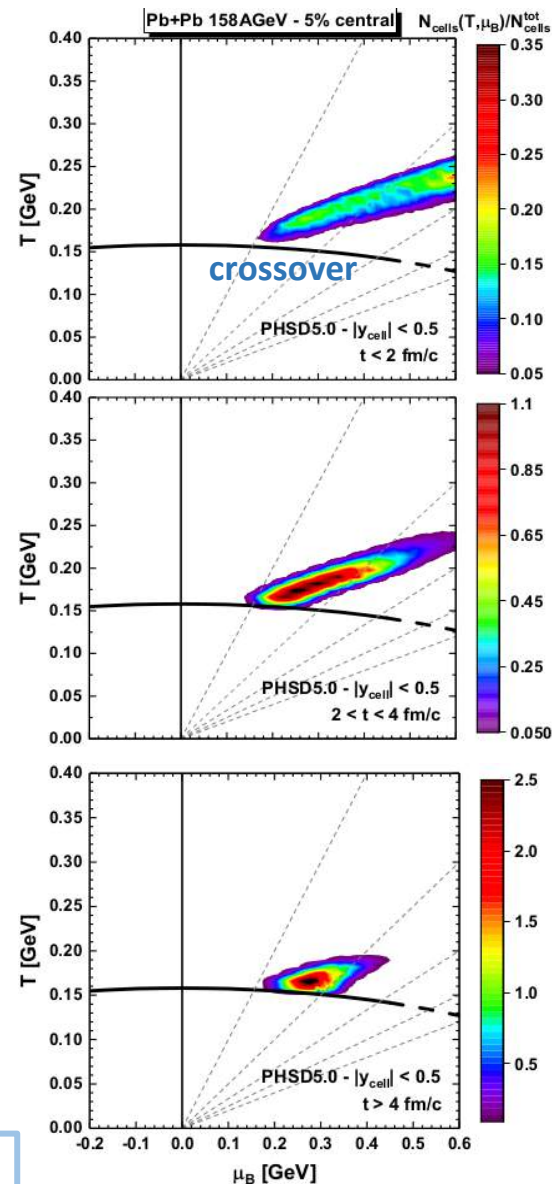
**Output:**

$T, \mu_B$

**T-profile in (x;y) &  $\mu_B$  profile in (x;y)**  
 at midrapidity ( $|y_{\text{cell}}| < 1$ ) at fixed times (1 and 4 fm/c)



**time evolution**



**Path through the phase diagram is not trivial and not localized**

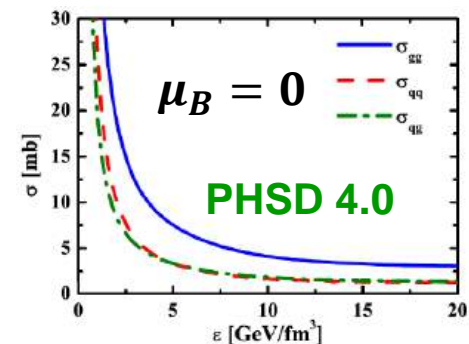
# Results for HICs from PHSD 4.0 and 5.0

Comparison between three different results:

- PHSD 4.0 : only  $\sigma(T)$  and  $\rho(T)$

$\sigma(T)$  – parton interaction cross sections

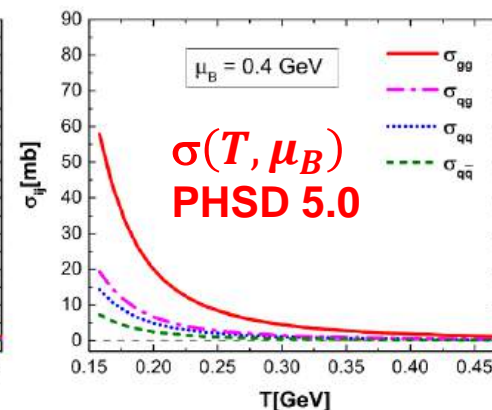
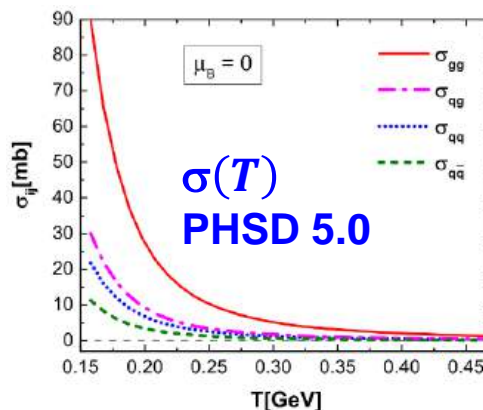
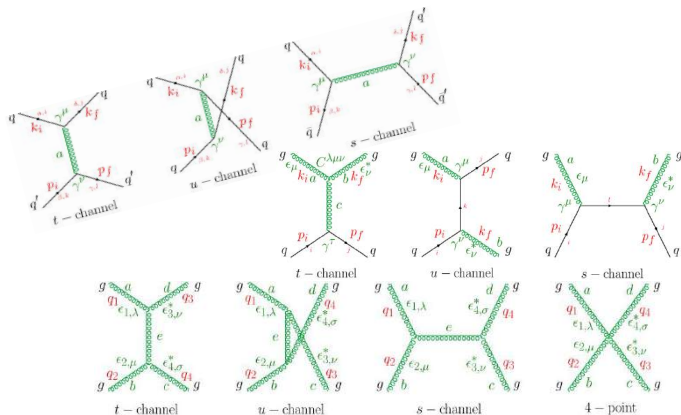
$\rho(T)$  – spectral function of partons (masses and widths)



**new PHSD 5.0 :  $\sqrt{s} + \mu_B$  + angular dependence of  $d\sigma/d\cos\theta$**

- PHSD 5.0 : with  $\sigma(\sqrt{s}, m_1, m_2, T, \mu_B = 0)$  and  $\rho(T, \mu_B = 0)$  with  $d\sigma/d\cos\theta$

- PHSD 5.0 : with  $\sigma(\sqrt{s}, m_1, m_2, T, \mu_B)$  and  $\rho(T, \mu_B)$  with  $d\sigma/d\cos\theta$

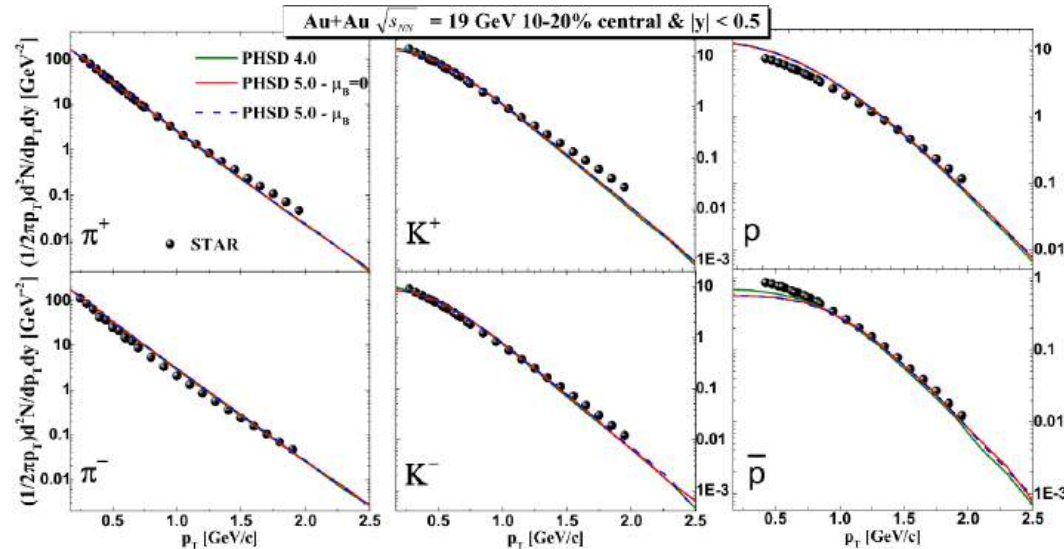
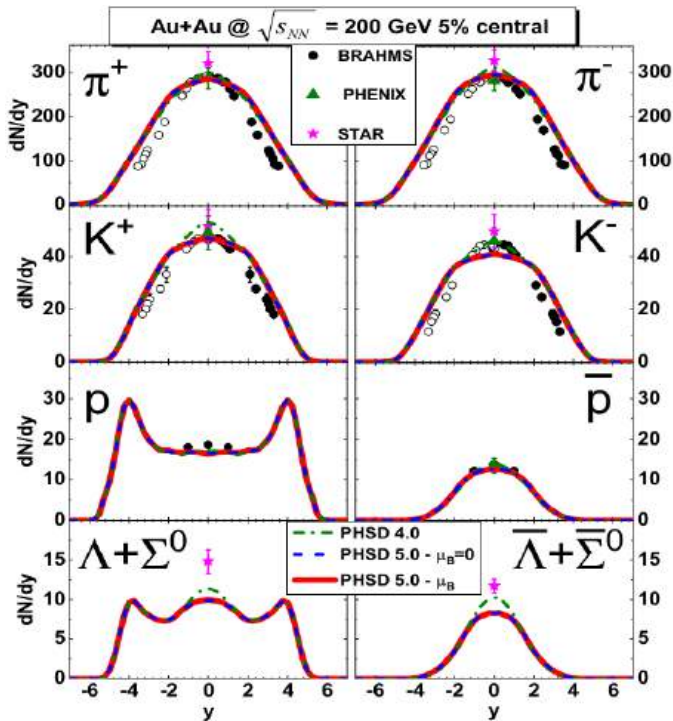
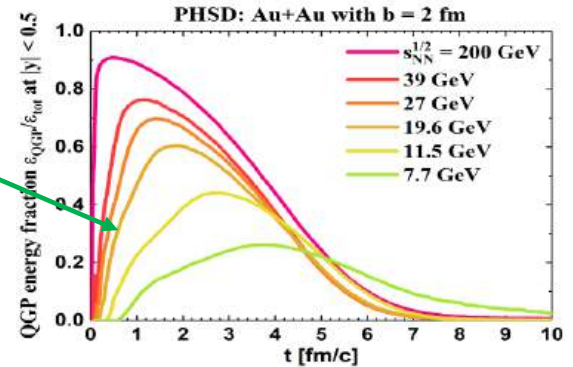


# Results for ( $\sqrt{s_{NN}} = 200 \text{ GeV} - 7 \text{ GeV}$ )

- No visible effects on  $p_T$ -spectra,  $dN/dy$  of  $\mu_B$ -dependence
- Small effect of the angular dependence of  $d\sigma/d\cos\theta$

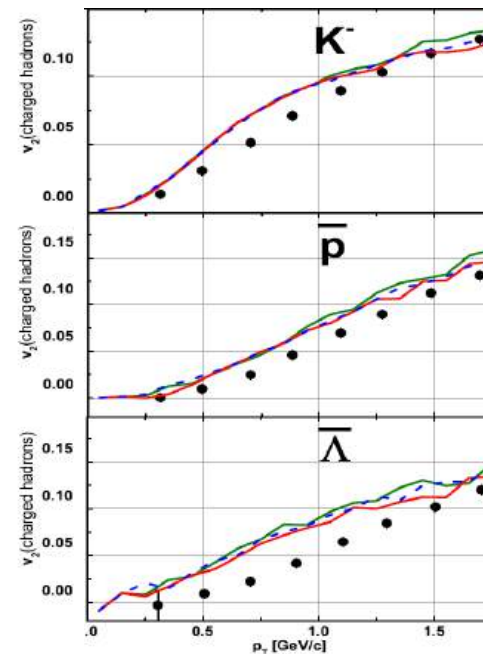
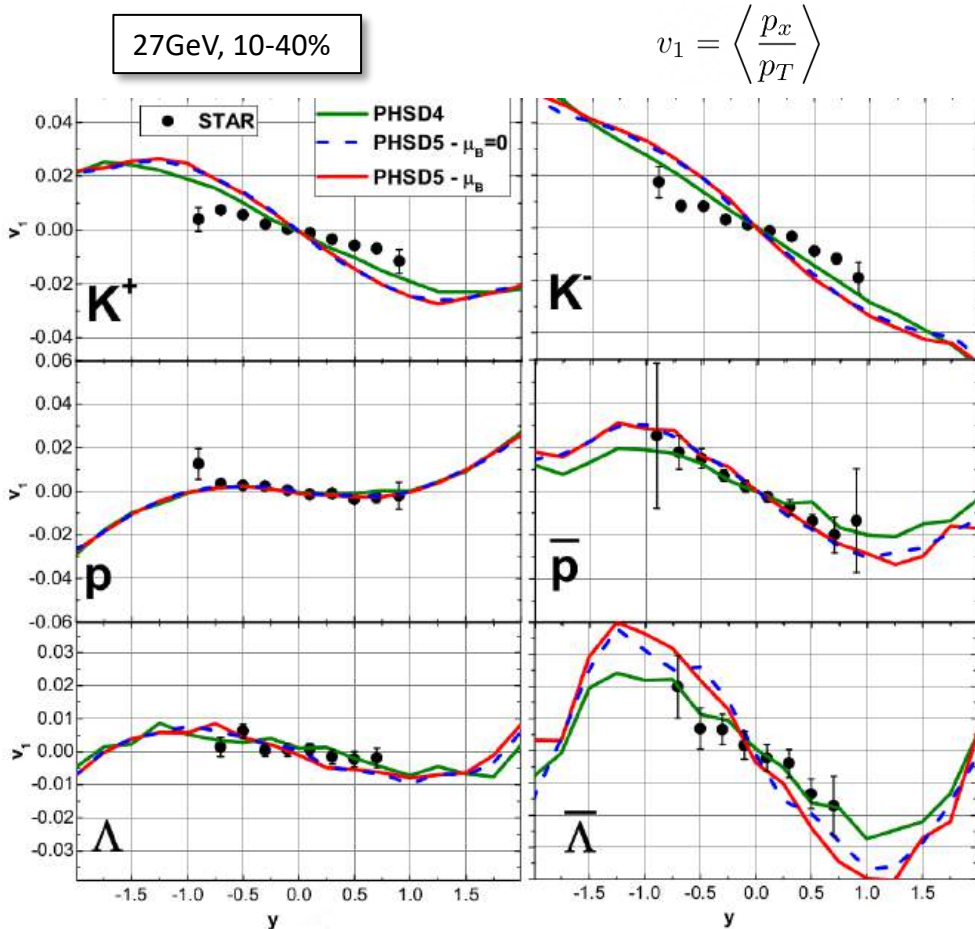
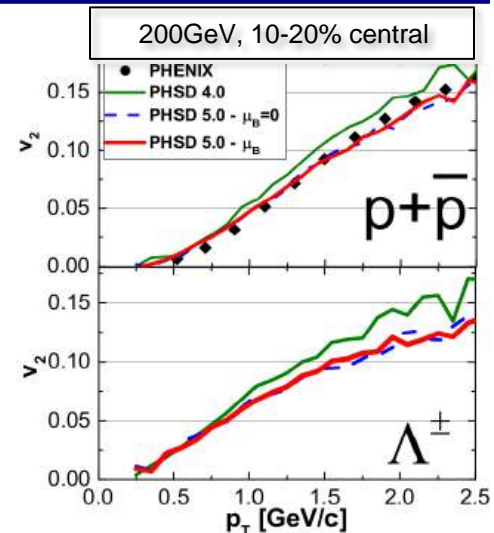
at high  $\sqrt{s_{NN}}$  - **low**  $\mu_B$

! QGP fraction is **small** at low  $\sqrt{s_{NN}}$

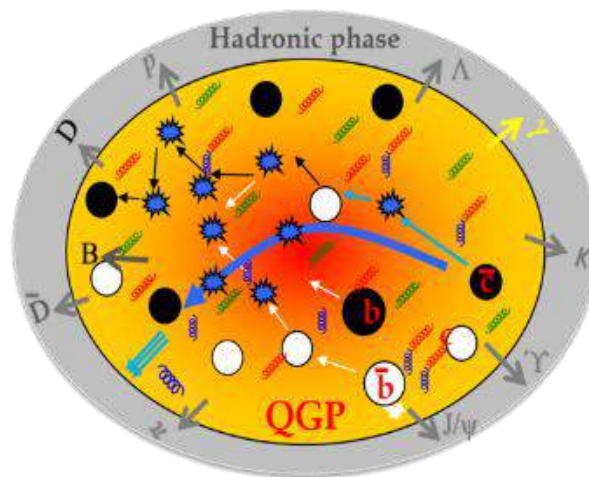


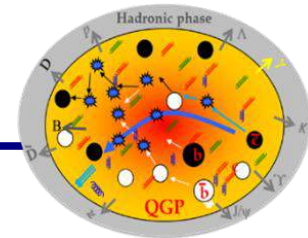
# Elliptic flow ( $\sqrt{s_{NN}} = 200 \text{ GeV} - 27 \text{ GeV}$ )

- **Weak  $\mu_B$  -dependence** – **small fraction of QGP or low  $\mu_B$**
- **Small effect of the angular dependence of  $d\sigma/d\cos\theta$**
- **Strong flavor dependence**



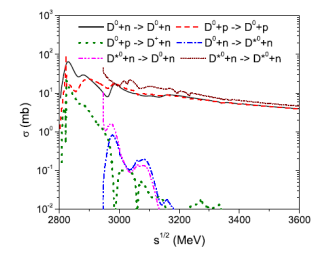
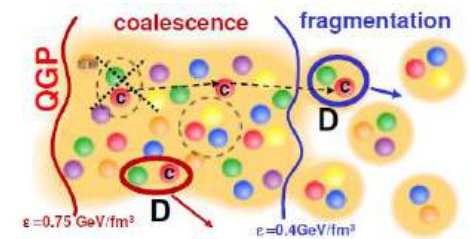
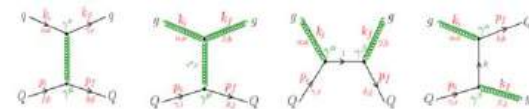
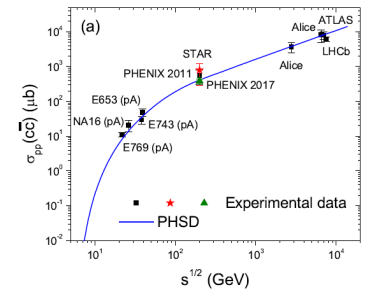
# Dynamics of heavy quarks – open charm and beauty (D/Dbar, B/Bbar) – in heavy-ion collisions





## Dynamics of heavy quarks in A+A :

- 1. Production of heavy (charm and bottom) quarks in initial binary collisions + shadowing and Cronin effects**
- 2. Interactions in the non-perturbative QGP – according to the DQPM: elastic scattering with off-shell massive partons  $Q+q \rightarrow Q+q$  → collisional energy loss**
- 3. Hadronization:  $c/\bar{c}$  quarks → D(D\*)-mesons: Dynamical hadronization scenario for heavy quarks :  
 coalescence with  $\langle r \rangle = 0.9$  fm & fragmentation  
 $0.4 < \varepsilon < 0.75$  GeV/fm<sup>3</sup>       $\varepsilon < 0.4$  GeV/fm<sup>3</sup>**
- 4. Hadronic interactions: D+baryons; D+mesons based on G-matrix and effective chiral Lagrangian approach with heavy-quark spin symmetry (>200 channels) (Juan Torres-Rincon, Laura Tolos)**



\* PHSD references on charm dynamics:

Taesoo Song et al., PRC 92 (2015) 014910, PRC 93 (2016) 034906, PRC 96 (2017) 014905  
 PRC 97 (2018) 064907; PRC 101 (2020) 044901; PRC 101 (2020) 044903

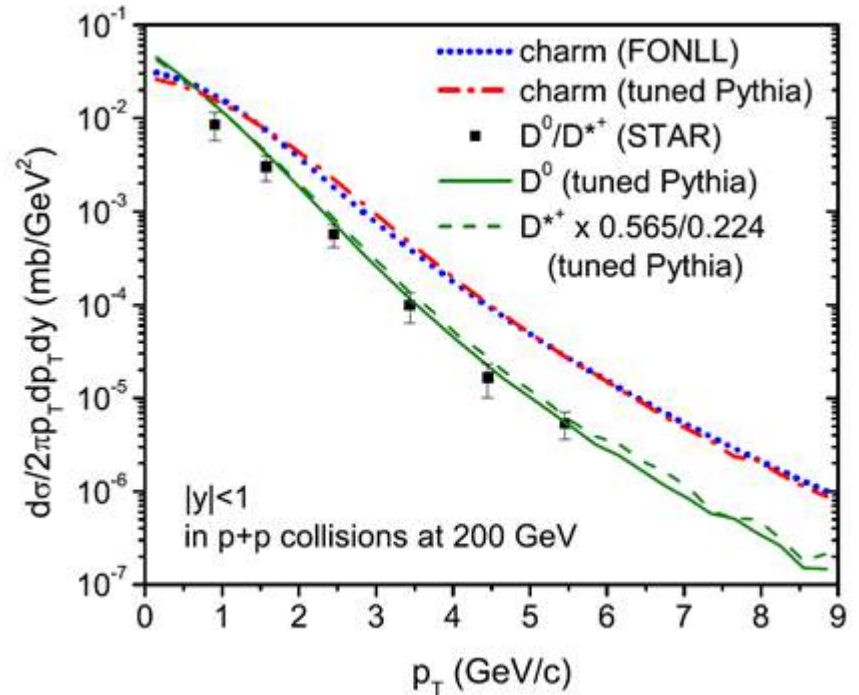
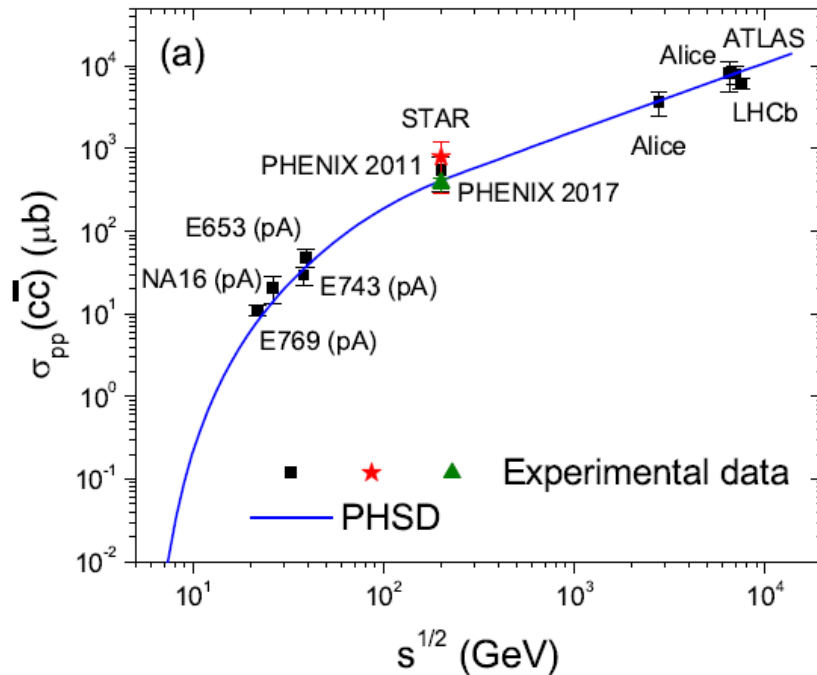


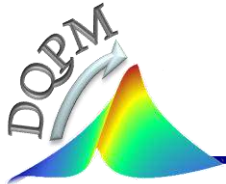
# Charm production in NN collisions

□ A+A: charm production in **initial NN binary collisions**: probability  $P = \frac{\sigma(cc\bar{c})}{\sigma_{NN}^{inel}}$

The **total cross section** for charm production in **p+p collisions**  $\sigma(cc)$

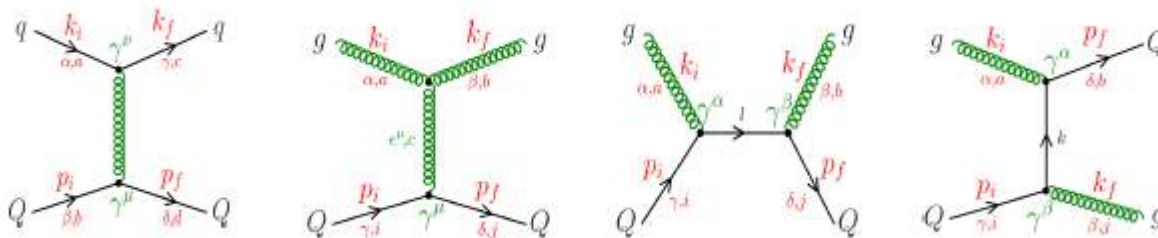
**Momentum distribution of heavy quarks**: use **'tuned' PYTHIA** event generator to reproduce **FONLL** (fixed-order next-to-leading log) results



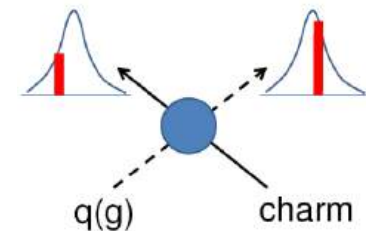


# Heavy quark scattering in the QGP (DQPM)

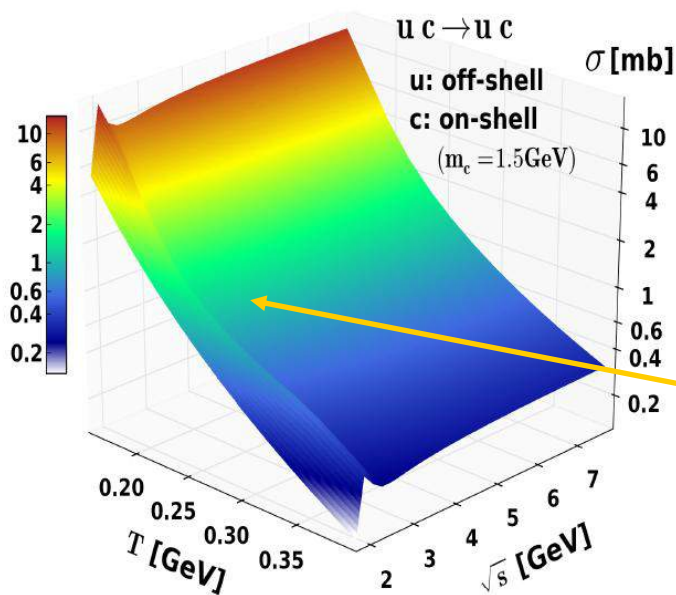
□ Elastic scattering with off-shell massive partons  $Q+q(g) \rightarrow Q+q(g)$



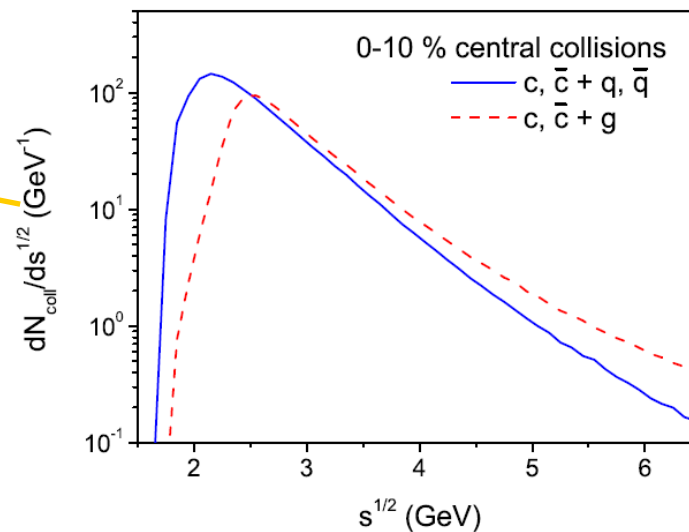
Non-perturbative QGP!

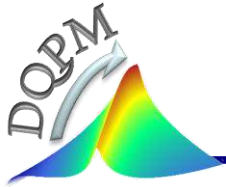


□ Elastic cross section  $uc \rightarrow uc$



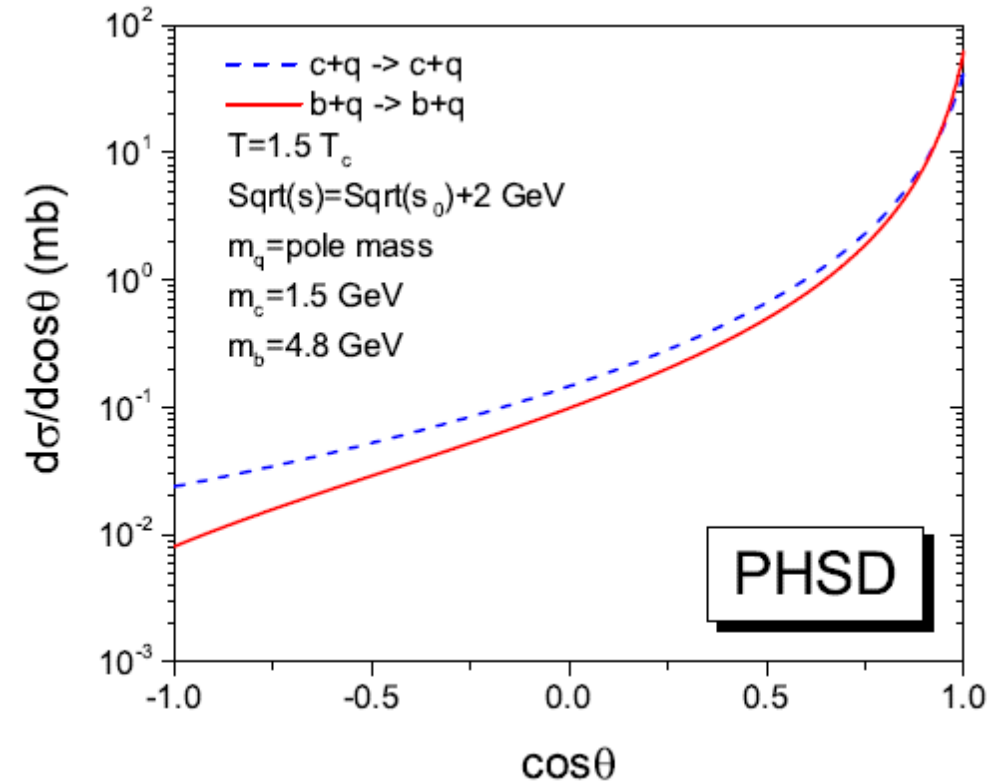
□ Distributions of  $Q+q$ ,  $Q+g$  collisions vs  $s^{1/2}$  in Au+Au, 200 GeV, 10% central





# Heavy quark scattering in the QGP

□ **Differential elastic cross section** for  $cq \rightarrow cq$ ,  $bq \rightarrow bq$  for  $s^{1/2} = s_0^{1/2} + 2\text{GeV}$  at  $1.5T_c$



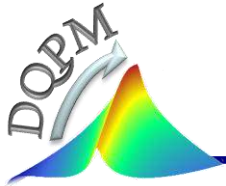
□ **DQPM - anisotropic angular distribution**

Note: pQCD - strongly forward peaked

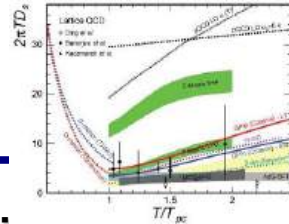
→ Differences between DQPM and pQCD :  
less forward peaked angular distribution  
leads to **more efficient momentum transfer**

→ Smaller number (compared to pQCD)  
of elastic scatterings with **massive**  
partons leads to a **larger energy loss**

! Note: **radiative energy loss** is **NOT** included yet in PHSD,  
it is expected to be **small** (at low  $p_T$ ) due to the large gluon mass in the DQPM

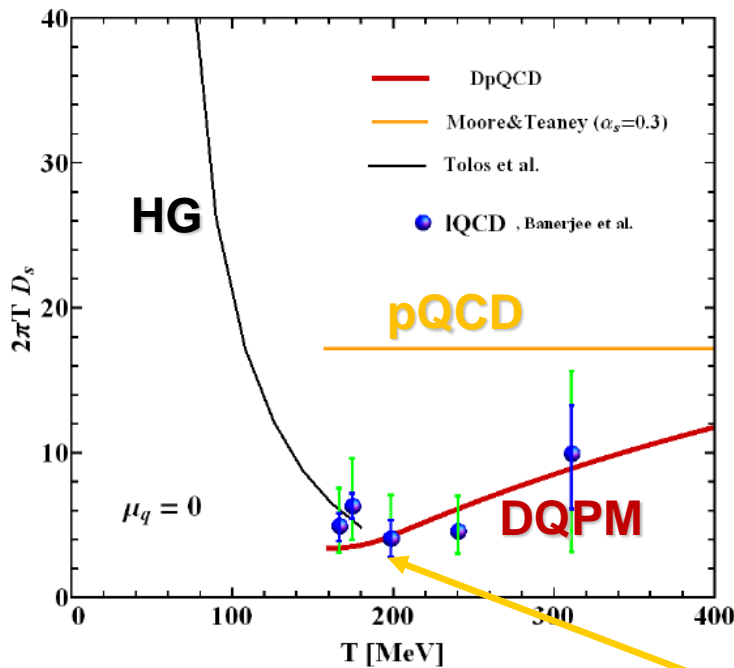


# Charm spatial diffusion coefficient $D_s$

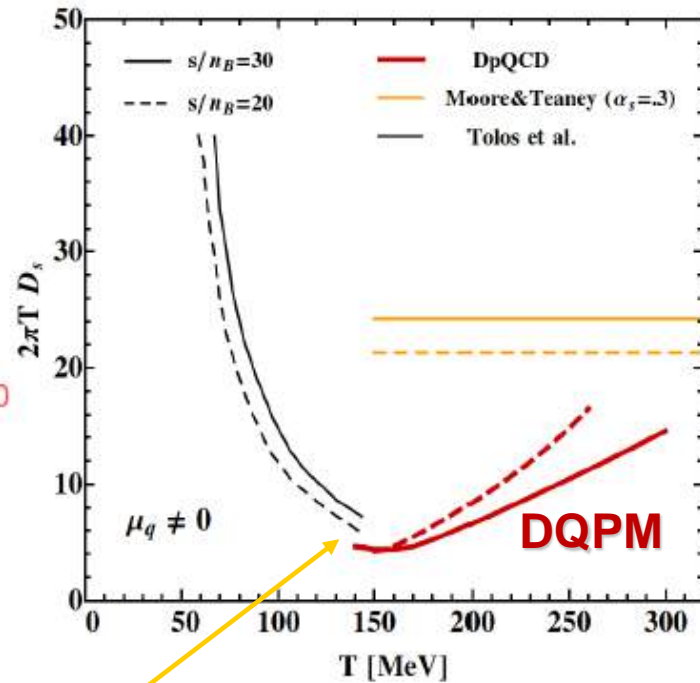


- $D_s$  for heavy quarks as a function of  $T$  for  $\mu_q=0$  and finite  $\mu_q$  assuming adiabatic trajectories (constant entropy per net baryon  $s/n_B$ ) for the expansion

$$D_s = \lim(\vec{p} \rightarrow 0) \frac{T}{M\eta_D} \quad \text{where } \eta_D = A/p ; A(p,T) = \text{drag coefficient}$$



$\Rightarrow$   
 $\mu_q \neq 0$

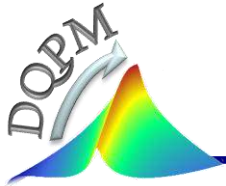


□  $T < T_c$  : hadronic  $D_s$

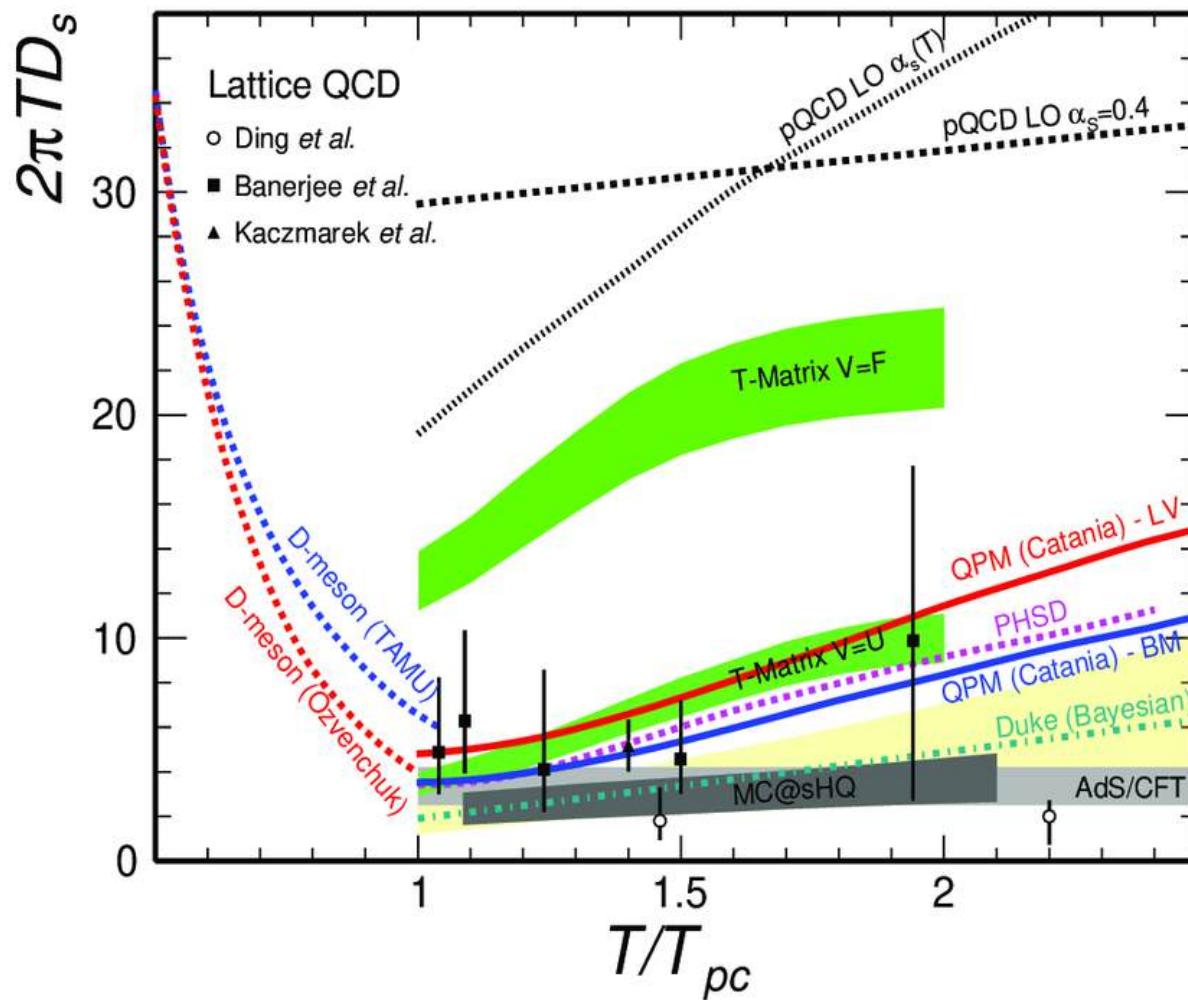
**→ Continuous transition at  $T_c$ !**

L. Tolos, J. M. Torres-Rincon, PRD 88 (2013) 074019  
V. Ozvenchuk et al., PRC90 (2014) 054909

H. Berrehrah et al, PRC 90 (2014) 051901, arXiv:1406.5322



# Charm spatial diffusion coefficient $D_s$

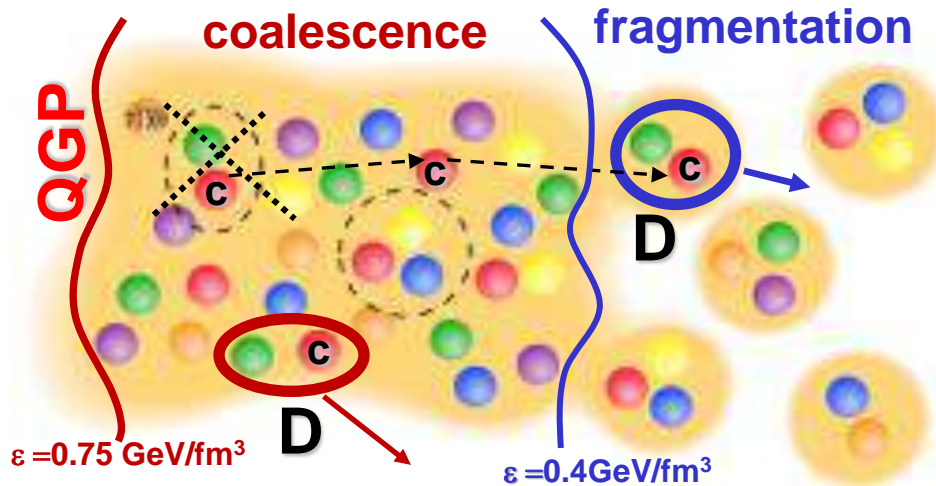


PHSD: if the local energy density  $\varepsilon \rightarrow \varepsilon_C \rightarrow$  hadronization of heavy quarks to hadrons

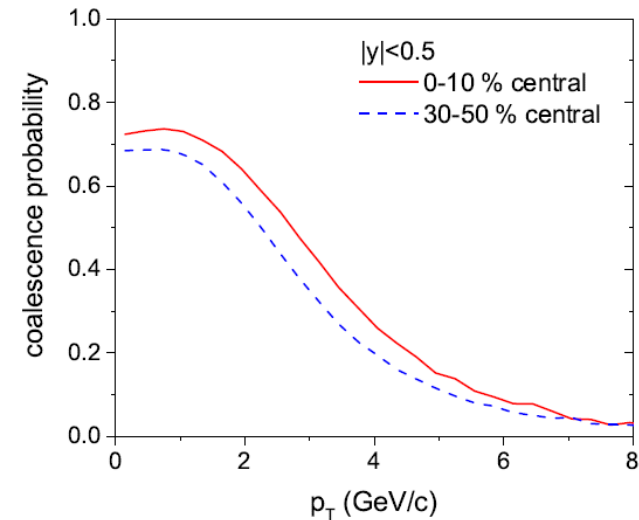
T. Song et al., PRC 93 (2016) 034906

## Dynamical hadronization scenario for heavy quarks :

coalescence with  $\langle r \rangle = 0.9$  fm & fragmentation  
 $0.4 < \varepsilon < 0.75$  GeV/fm<sup>3</sup>       $\varepsilon < 0.4$  GeV/fm<sup>3</sup>



## Coalescence probability in Au+Au at LHC



Coalescence probability for  $c + \bar{q} \rightarrow D$

$$f(\rho, \mathbf{k}_\rho) = \frac{8g_M}{6^2} \exp \left[ -\frac{\rho^2}{\delta^2} - \mathbf{k}_\rho^2 \delta^2 \right]$$

where  $\rho = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2)$ ,  $\mathbf{k}_\rho = \sqrt{2} \frac{m_2 \mathbf{k}_1 - m_1 \mathbf{k}_2}{m_1 + m_2}$

Width  $\delta \leftarrow$  from root-mean-square radius of meson  $\langle r \rangle$ :

$$\langle r^2 \rangle = \frac{3}{2} \frac{m_1^2 + m_2^2}{(m_1 + m_2)^2} \delta^2$$

Degeneracy factor :  $g_M = 1$  for D, = 3 for  $D^* = D^*_0(2400)^0$ ,  $D^*_1(2420)^0$ ,  $D^*_2(2460)^{0\pm}$



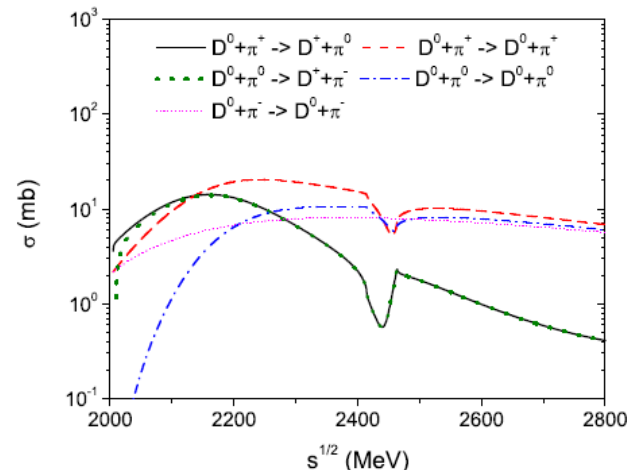
# D-meson scattering in the hadronic phase

## 1. D-meson scattering with mesons

L. M. Abreu, D. Cabrera, F. J. Llanes-Estrada, J. M. Torres-Rincon, *Annals Phys.* **326**, 2737 (2011)

**Model: effective chiral Lagrangian approach with heavy-quark spin symmetry**

Interaction of  $D=(D^0, D^+, D^+_s)$  and  $D^*=(D^{*0}, D^{*+}, D^{*+}_s)$  with octet ( $\pi, K, Kbar, \eta$ )



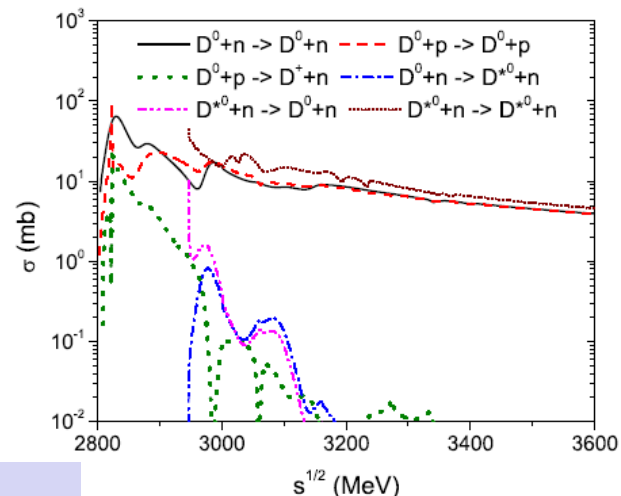
## 2. D-meson scattering with baryons

C. Garcia-Recio, J. Nieves, O. Romanets, L. L. Salcedo, L. Tolos, *Phys. Rev. D* **87**, 074034 (2013)

**Model: G-matrix approach:** interactions of  $D=(D^0, D^+, D^+_s)$  and  $D^*=(D^{*0}, D^{*+}, D^{*+}_s)$  with nucleon octet  $J^P=1/2^+$  and Delta decuplet  $J^P=3/2^+$

Unitarized scattering amplitude  $\rightarrow$  solution of coupled-channel **Bethe-Salpeter equations:**

$$T = T + VGT$$



$\rightarrow$  Strong **isospin dependence** and complicated structure (due to the resonance coupling) of D+m, D+B cross sections!

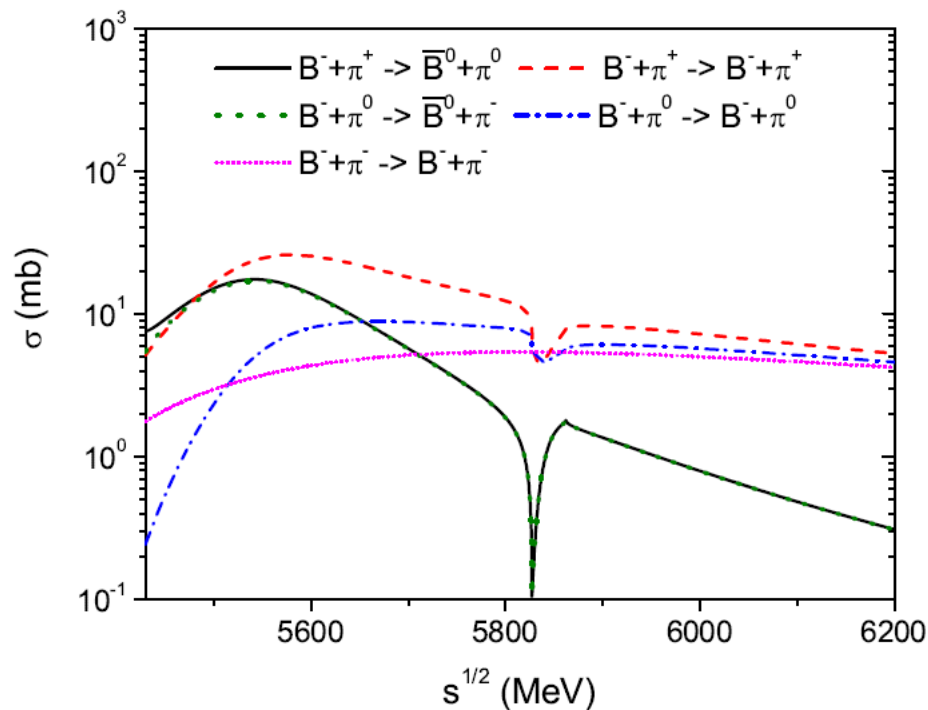


# B-meson scattering in the hadron gas

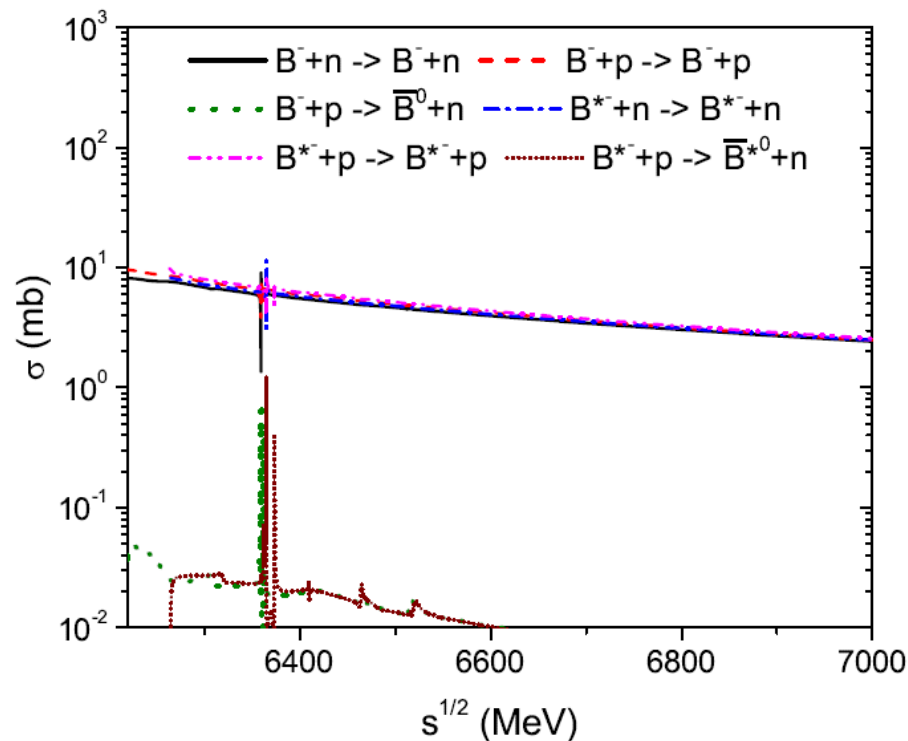
L. Tolos and J. M. Torres-Rincon, Phys. Rev. D 88, 074019 (2013)

J. M. Torres-Rincon, L. Tolos and O. Romanets, Phys. Rev. D 89, 074042 (2014)

## 1. B-meson scattering with mesons



## 2. B-meson scattering with baryons



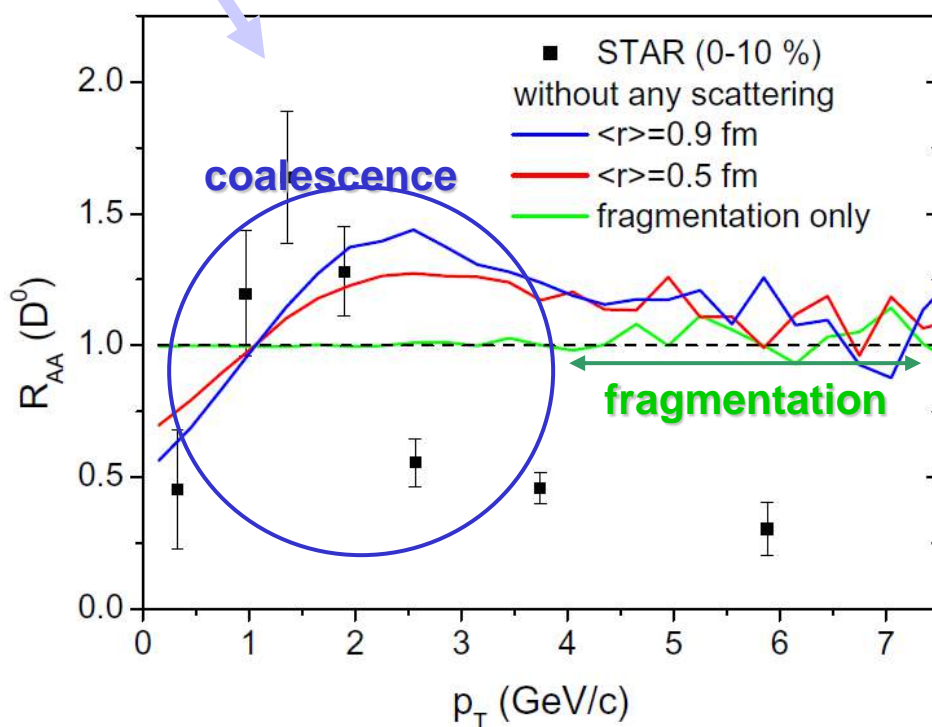
➤ **>200 hadronic channels** → implemented in the PHSD (by Taesoo Song)



# $R_{AA}$ at RHIC - coalescence vs fragmentation

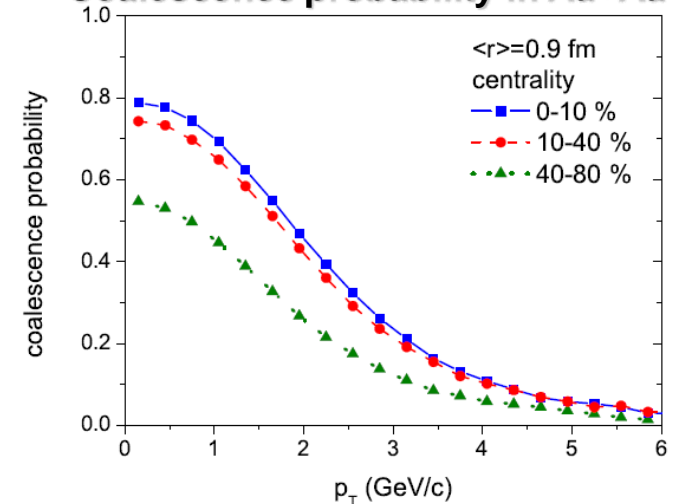
Influence of hadronization scenarios: coalescence vs fragmentation

**! Model study:** without any rescattering (partonic and hadronic)



$$R_{AA}(p_T) \equiv \frac{dN_D^{Au+Au}/dp_T}{N_{binary}^{Au+Au} \times dN_D^{P+P}/dp_T}$$

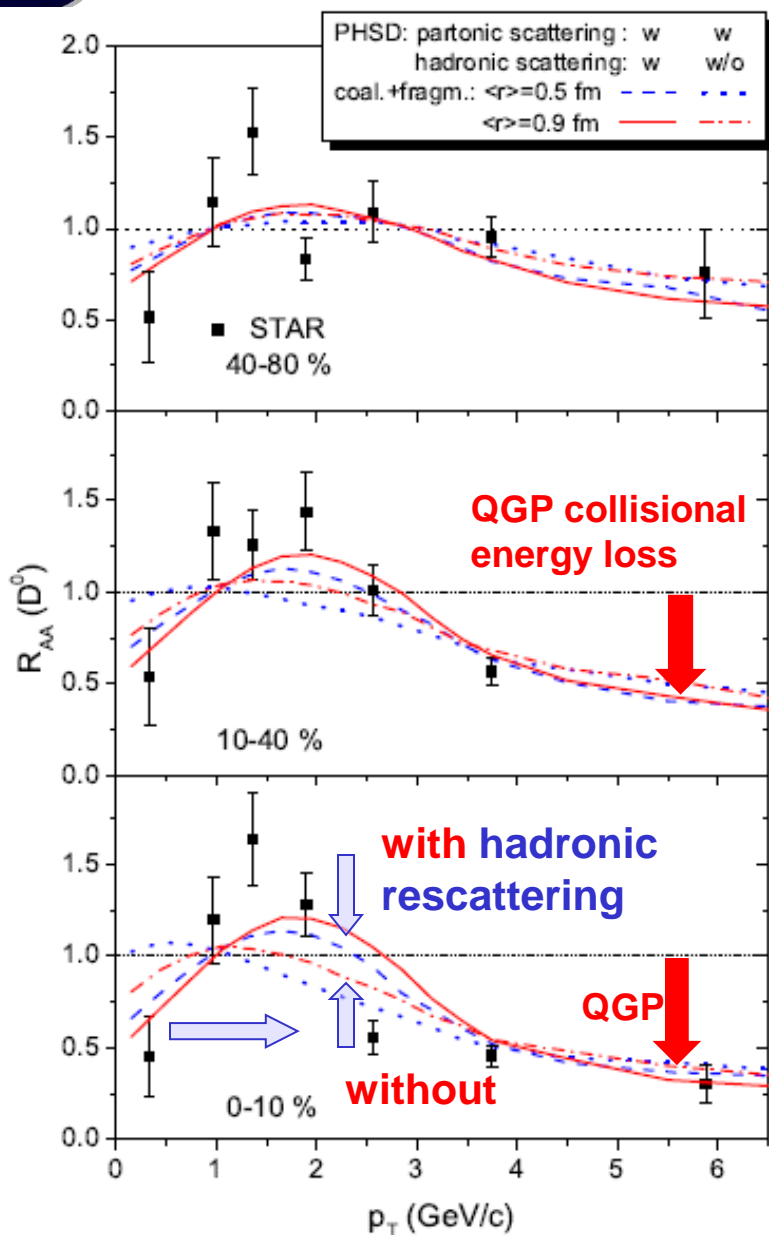
Coalescence probability in Au+Au



- Expect: no scattering:  $R_{AA}=1$
- Hadronization by fragmentation only (as in pp)  $\rightarrow R_{AA}=1$
- Coalescence (not in pp!) shifts  $R_{AA}$  to larger  $p_T \rightarrow$  'nuclear matter' effect
- The height of the  $R_{AA}$  peak depends on the balance: coalescence vs. fragmentation



# $R_{AA}$ at RHIC: hadronic rescattering



## Influence of hadronic rescattering:

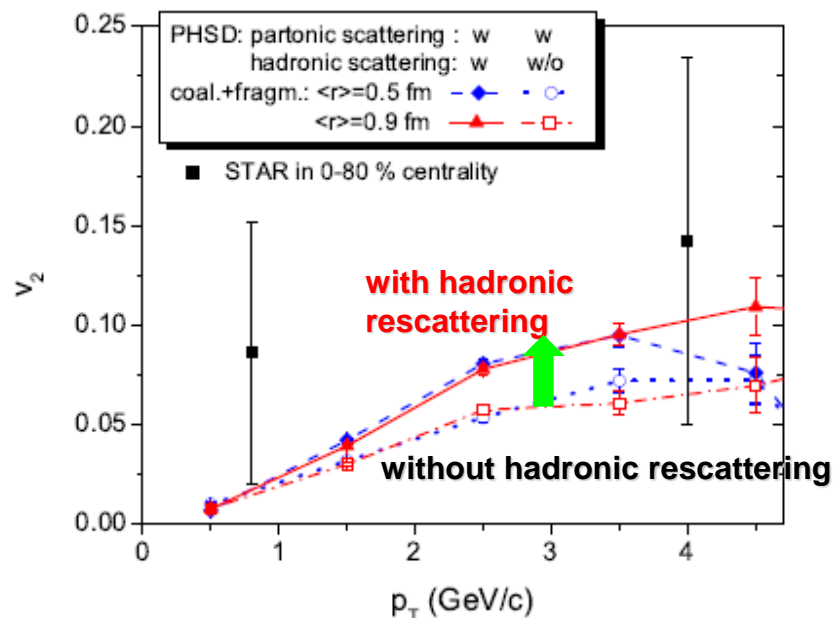
Central Au+Au at  $s^{1/2} = 200$  GeV :

$N(D, D^*) \sim 30$

$N(D, D^* + m) \sim 56$  collisions

$N(D, D^* + B, Bbar) \sim 10$  collisions

→ each  $D, D^*$  makes  $\sim 2$  scatterings with hadrons



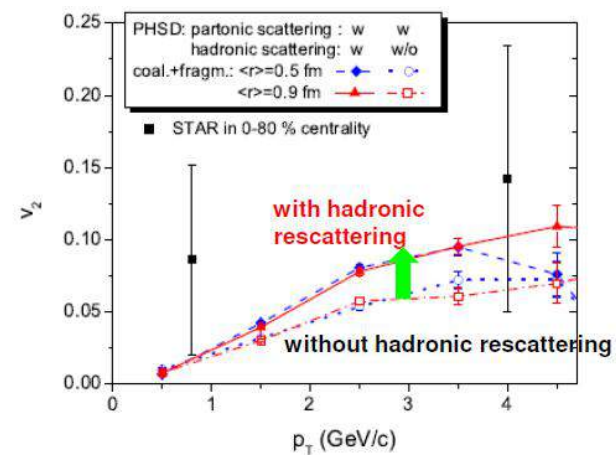
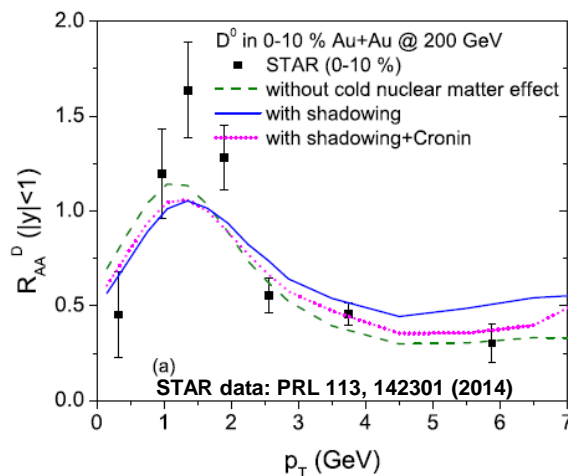
- Hadronic rescattering moves  $R_{AA}$  peak to higher  $p_T$ !
- substantially increases  $v_2$  at larger  $p_T$



# PHSD vs charm observables at RHIC

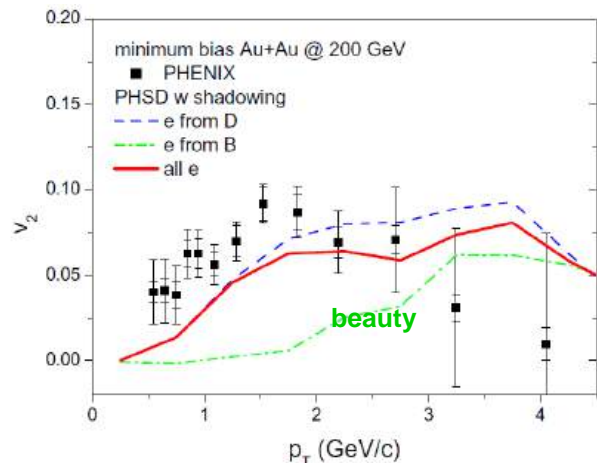
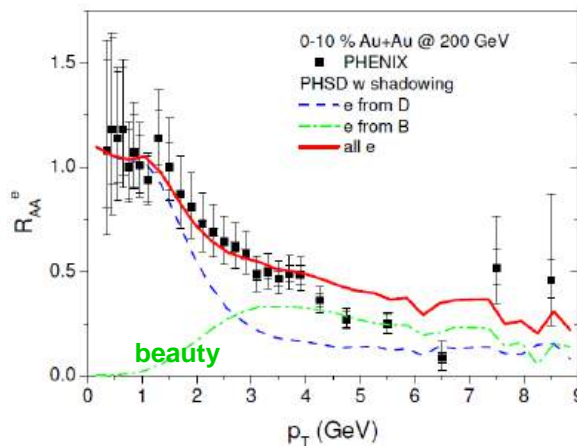
## STAR

$R_{AA}$  and  $v_2$  vs  $p_T$   
from  $D^0$ -mesons  
in Au+Au @ 200 GeV →



## PHENIX

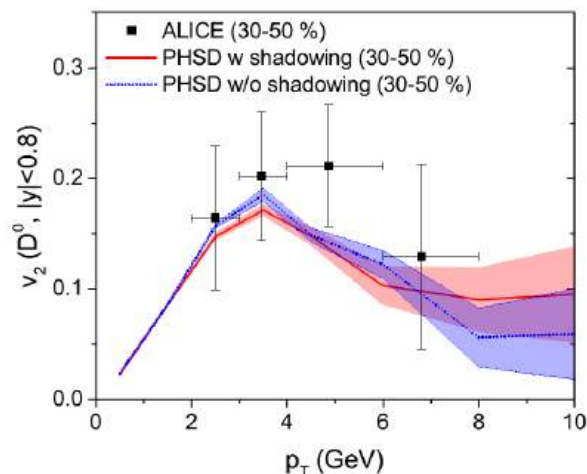
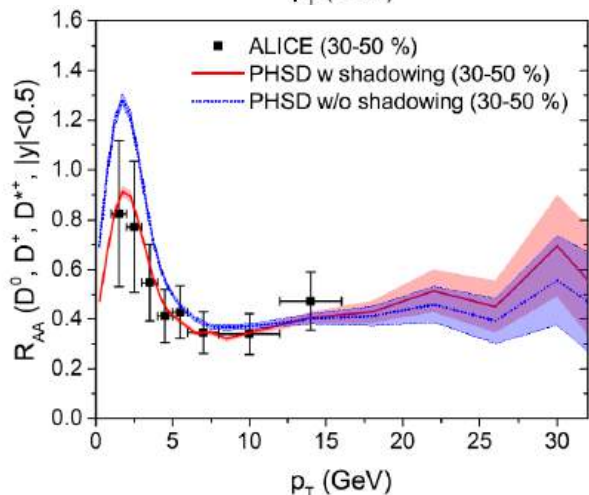
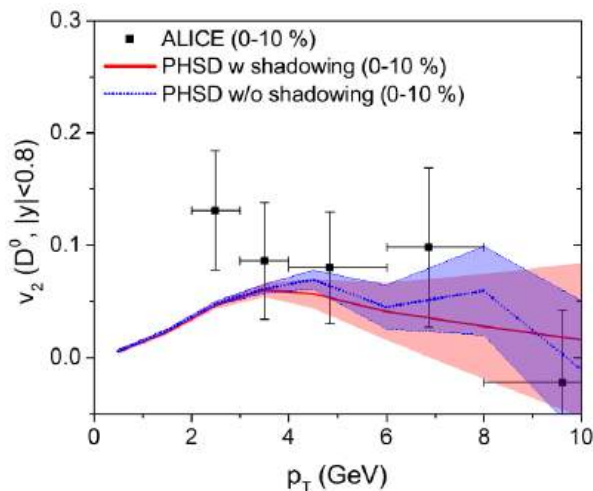
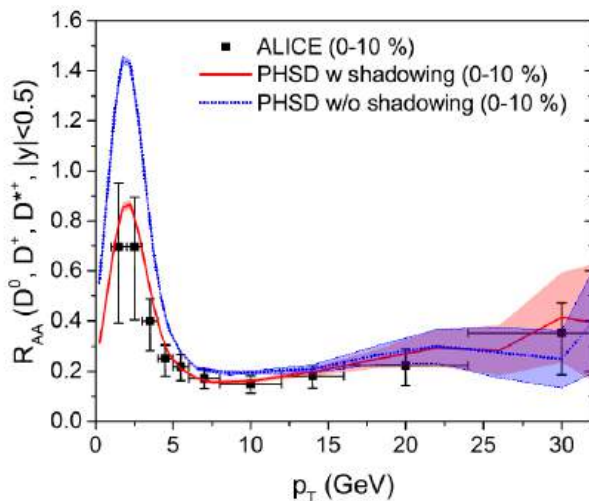
$R_{AA}$  and  $v_2$  vs  $p_T$   
from single electrons  
in Au+Au @ 200 GeV →



- The exp. data for the  $R_{AA}$  and  $v_2$  are described in the PHSD by **QGP collisional energy loss** due to elastic scattering of charm quarks with massive quarks and gluons in the QGP
- + by the **dynamical hadronization scenario** „coalescence & fragmentation“
- + by **strong hadronic interactions** due to resonant elastic scattering of  $D, D^*$  with mesons and baryons
- **Feed back from beauty** contribution becomes dominant for single electrons  $R_{AA}$  and  $v_2$  at  $p_T > 3$  GeV



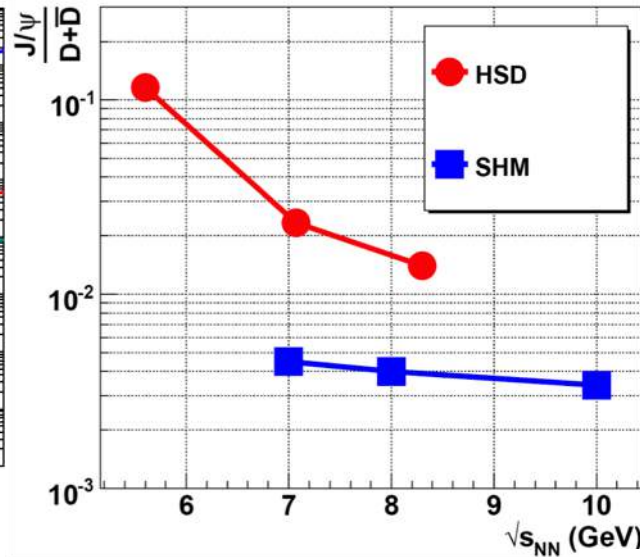
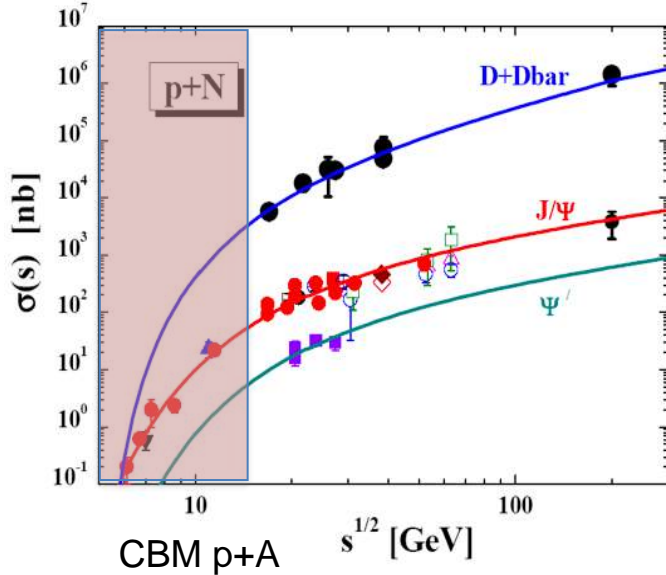
# Charm $R_{AA}$ at LHC: PHSD vs ALICE



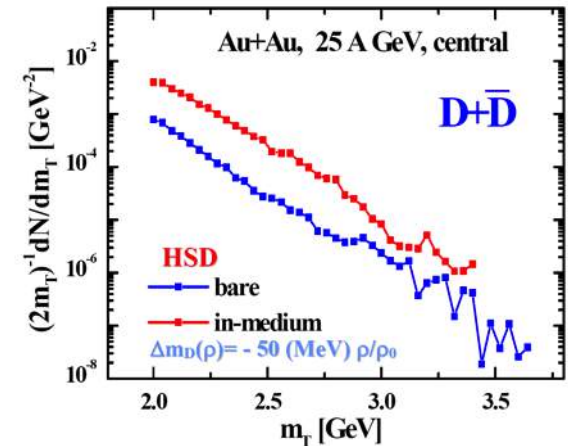
- in PHSD the energy loss of D-mesons at high  $p_T$  can be dominantly attributed to partonic scattering
- Shadowing effect suppresses the low  $p_T$  and slightly enhances the high  $p_T$  part of  $R_{AA}$
- Hadronic rescattering moves  $R_{AA}$  peak to higher  $p_T$ ; increases  $v_2$

# Why Open Charm with CBM?

O. Lynnek et al., Nucl. Phys. A 786 (2007) 183



- elementary cross section experimentally unknown
- no data in A+A below RHIC energies
- open / hidden charm seems promising observable to distinguish phases
- in-medium modification of D masses?

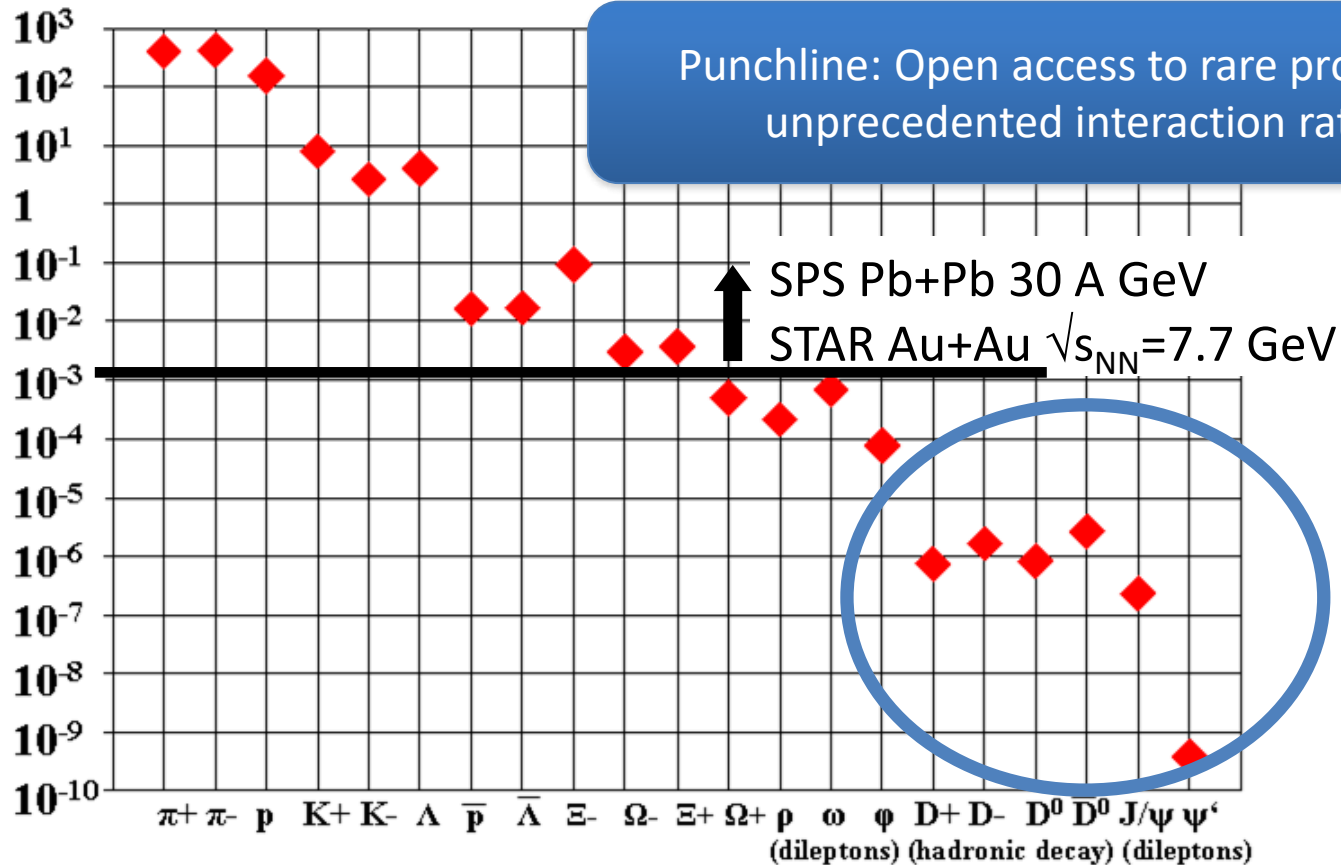


Slide from

# Experimental challenges: Rare probes

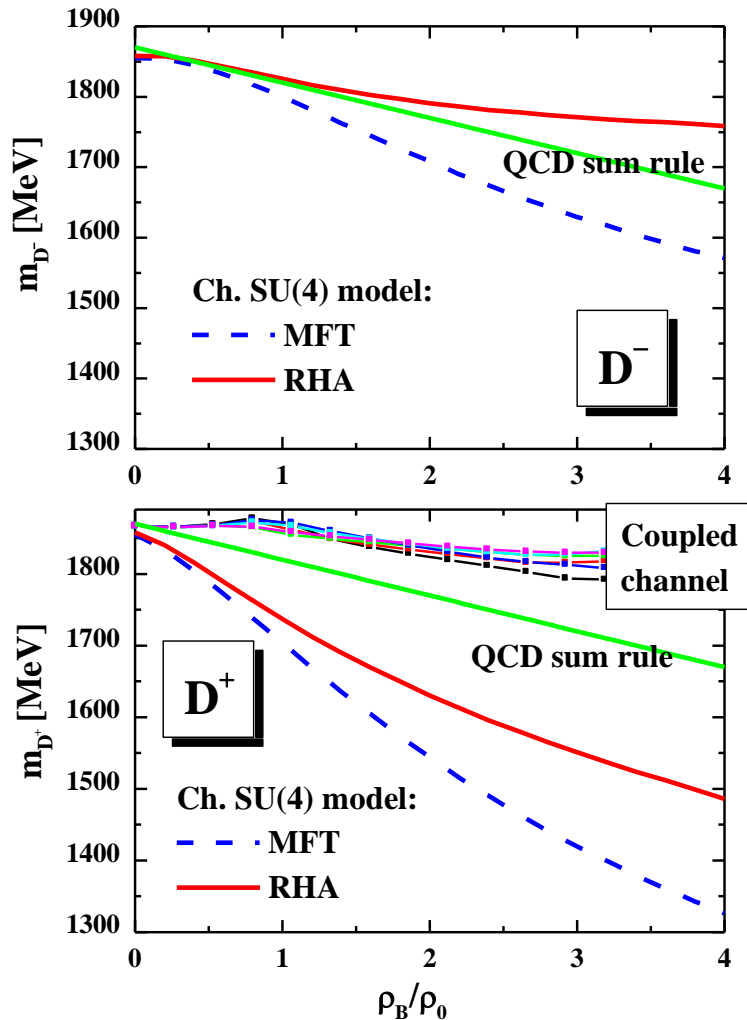
particle multiplicity  
× branching ratio

min. bias Au+Au collisions at 25 AGeV  
(from HSD and thermal model)

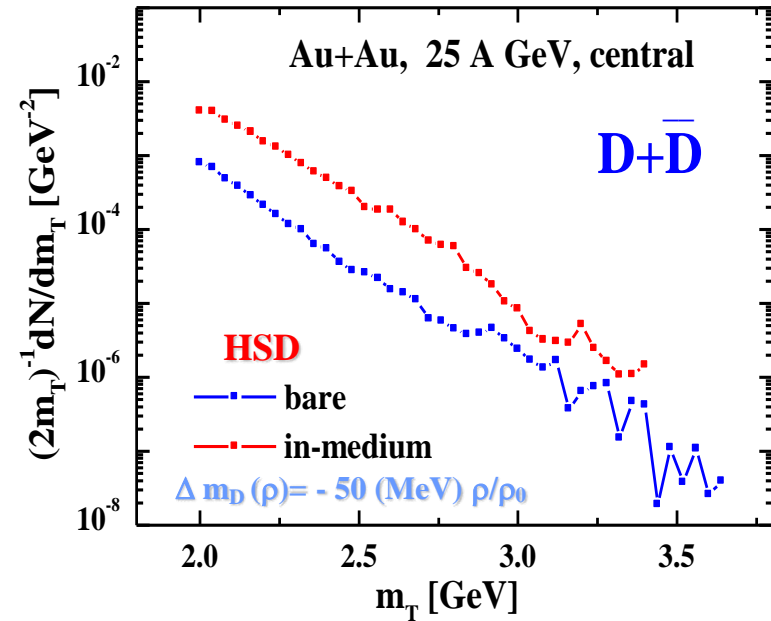


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# D/Dbar-mesons: in-medium effects

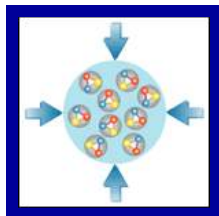


Ch. SU(4): A. Mishra et al., PRC69 (2004) 015202  
 QCD sum rule: Hayashigaki, PLB487 (2000) 96  
 Coupled channel: Tolos et al., EPJ C43 (2005) 761

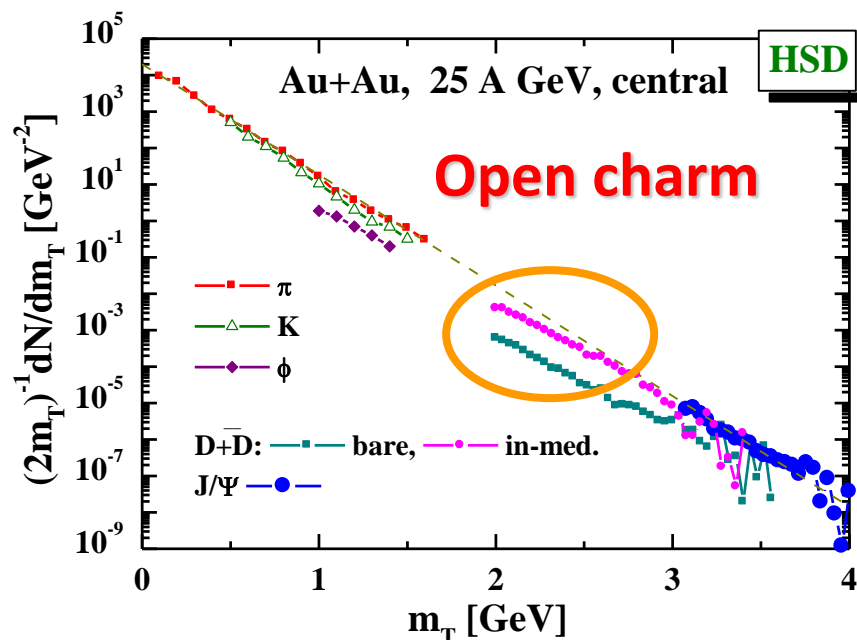
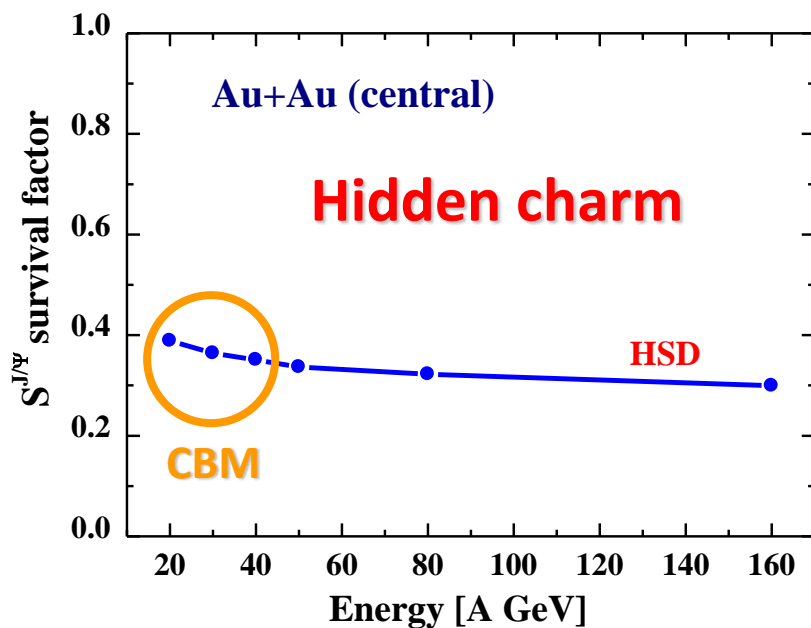


- **Dropping D-meson masses with increasing light quark density might give a large enhancement of the open charm yield at 25 A GeV !**
- **Charmonium suppression increases for dropping D-meson masses!**

**FAIR (CBM)**



# Open and hidden charm – HSD predictions for CBM



## Open charm:

- **without medium effects:** suppression of D-meson spectra by factor  $\sim 10$  relative to the global  $m_T$ -scaling
- **with medium effects:** restoration of the global  $m_T$ -scaling for the mesons

## Hidden charm:

J/ $\psi$  suppression due to comover absorption at FAIR is lower than at SPS